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FAILING FIRMS AND SUCCESSFUL ENTREPRENEURS: SERIAL ENTREPRENEURSHIP AS A SIMPLE MACHINE

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ABSTRACT

Given what we already know about firm success and failure, namely that most firms fail, can we say anything about the possible success or failure of entrepreneurs? In this paper we argue that irrespective of what we believe the failure rate of firms to be, we can still rigorously understand entrepreneurial success/failure and derive useful prescriptions to improve success rates of *entrepreneurs*. Particularly, entrepreneurs can use Bayesianism as a control mechanism, instead of as an inference engine.

"Most firms fail," appears to be a consensus among entrepreneurship scholars and practitioners alike, even when they disagree on the actual proportions (Aldrich & Martinez, 2001; Fichman & Levinthal, 1991; Hannan & Freeman, 1984; Low & MacMillan, 1988; Stinchcombe, 1965). Estimates of firm success rates range from the highly disputed but optimistic 44% of Kirchhoff (1997) to the widely acknowledged one in ten of the National Venture Capitalists Association. Under these circumstances, economists such as Arrow are not easily refuted in their claims about the irrelevancy of business school programs that profess to "teach" entrepreneurship¹: *Are we trying to isolate a claim that some particular set of individuals with certain characteristics or particular set of institutions create -- distinguish the successes and the failures? And this introduces me to what I call the null hypothesis: That there is no such thing.*

Such a null hypothesis begs the question as to why any entrepreneur would ever start a firm, to say nothing of the serial entrepreneur who starts several, both before and after successes and failures. To that the economist normally replies either that the entrepreneur is extraordinarily risk loving, or that he or she operates under the illusion that the expected value of the payoff (estimated expected return multiplied by their subjective probability of success) is high enough to spur entry – or both. There is credible empirical evidence that the former explanation based on a supra-normal preference for risk, cannot be justified. Entrepreneurs have been shown to range all over the risk preference spectrum and the distribution may even be skewed toward risk aversion rather than otherwise (Brockhaus, 1980; Palich & Bagby, 1995; Sarasvathy, Simon & Lave, 1997).

¹ JBV Transcription of Report on the seminar on research perspectives in entrepreneurship. Journal of Business <u>Venturing</u>, 15(1).

As for the latter (that the expected payoff is high enough to spur entry), there are no studies on how the entrepreneur estimates his or her subjective probability of success or failure. Nor are there any studies that indicate how they *ought* to estimate such a probability. One reason for this omission could be the extraordinary difficulties in even estimating the rates of *firm* successes and failures. Problems range from hindrances in data collection especially about failures, to incompatibilities in the complex taxonomy of firm characteristics that make definitions of rates of success or failure meaningless. For example, how can one compare the success of a bed and breakfast in Vermont with that of a bio-tech startup in Seattle? Therefore, the overall practice of the extensive literature on estimating rates of firm success/failure is to unwittingly or explicitly equate the expected success rate of firms with the expected success rate of entrepreneurs.

This leads us to the central question of this paper: *Given what we already know about firm success and failure, namely that most firms fail, can we say anything about the possible success or failure of entrepreneurs?* In the following pages we argue that irrespective of what we might believe the failure rate of firms to be, we can still rigorously understand important relationships between entrepreneurial success and failure and derive useful prescriptions to improve the success rates of *entrepreneurs*. We begin our investigation by reviewing three streams of literature to summarize what we know about entrepreneurial success. Next we examine the space of entrepreneurs as distinct from the space of firms and discuss the transformation of measures between the two, as described by Bayes' formula. Critical to our exposition is the reinterpretation of Bayes' formula in terms of *control* rather than *prediction*, that is, as a tool for *shaping events* rather than for *updating beliefs*.

WHAT WE KNOW ABOUT ENTREPRENEURIAL SUCCESS/FAILURE

Success rates of firms and entrepreneurs have been studied extensively by a variety of researchers under a number of rubrics such as: *firm formation and entry* (by scholars in industrial organization); *organizational founding and survival* (by population ecologists and organizational theorists); and, *entrepreneurial success and failure* (by entrepreneurship researchers). We now examine each of these areas and summarize their findings to show that all of them either confound the spaces of entrepreneurs and firms, or focus exclusively on the space of firms.

From Studies of Industrial Organization

Following a plea by Edwin Mansfield (1962: 1023), to encourage econometric studies of the birth, growth, and death of firms, a slew of industrial organization scholars began studying the process of entry with a view to understanding its determinants as well as its impact on market performance. In an excellent review of this stream of research, Geroski (1995) summarizes the results as a series of stylized facts that are generally agreed upon by scholars in the area. For our particular purposes in this paper, the key facts from this body of work are: (a) While entry is common, survival is not. In other words, while large numbers of firms enter most markets in most years, survival of new entrants, especially de novo entrants, is low; and, (b) Most markets are subject to enormous waves or bursts of entry in the early stages of their life cycles.

From Studies of Population Ecology of Organizations

The above two results culled from industrial organization are independently supported (at least partially) by organization theorists who use an evolutionary and/or population ecology perspective (Aldrich & Fiol, 1994). Population ecologists have found that success rates of

organizations are age dependent. As concisely summarized by Henderson (1999), this literature does not always agree on the exact relationship between the age of a firm and its probability of success or failure. While some stress the liability of newness as a factor of firm failure (e.g. Stinchcombe, 1965; Hannan & Freeman, 1984), others argue that there is an early window of survival due to the initial stock of assets acquired at founding after which the liability of adolescence takes over and reduces the life expectancy of firms (Bruderl & Schlussler, 1990; Fichman & Levinthal, 1991). But besides the high probability of infant (or adolescent) mortality, this literature also finds a high probability of failure due to old age when firms tend to become highly inertial and misaligned with their environments (Baum, 1989; Barron, West & Hannan, 1994).

Neither the industrial organization literature, nor the one based on population ecology addresses the success or failure rates of *entrepreneurs*.

From Entrepreneurship Research

Entrepreneurship scholars do worry about entrepreneurs as well as firms. All the same, it is in this literature that the greatest confounding between firms and entrepreneurs occurs. For example, there is a rather large stream of effort in this literature devoted to the traits and characteristics of entrepreneurs and how they affect firm performance. In a comprehensive review of this stream, Gartner (1988) identified a number of studies starting around the middle of the twentieth century that focused on the personality of the entrepreneur as a predictor of firm success. He argued for the futility of the traits approach since it sought to separate "the dancer from the dance" and in over three decades did not result in any clear understanding of the phenomena concerned with firm creation. Although the traits approach has since been largely abandoned, recent studies have turned to a more sophisticated understanding of the cognitive biases of entrepreneurs and their ability to garner human and social capital as predictors of firm success. Examples include Baron (2000), Bates (1990), and Busenitz & Barney (1997). Also interesting are studies such as Gimeno, et. al. (1997) that relate firm survival to factors other than objective measures of firm performance. In particular they find that subjective thresholds of performance based on human capital characteristics of entrepreneurs (such as alternative employment opportunities, psychic income from entrepreneurship, and cost of switching to other occupations) result in firm survival even in the case of so-called "underperforming" firms. All the same the focus on the personality of the entrepreneur as a predictor of firm success is not quite dead, as is evidenced by Brandstaetter (1997), and Miner (1997).

The primary reason for the paucity of evidence about the success and failure of *entrepreneurs* as distinct from firms consists in the fact that while evidence on failed firms is hard enough to obtain (the data usually disappear along with the demise of the firm), evidence on failed entrepreneurs is well nigh impossible to come by. People just simply do not walk around with business cards that say "failed entrepreneur." Most founders of failed firms either dust themselves off and go on to start other firms or are serial entrepreneurs who have *previously* been successful. Both these groups tend not to mention their failed firms except as part of uplifting anecdotes in public speeches, after the fact. The few truly "failed entrepreneurs" seemingly disappear off the face of the economy forever leaving us, entrepreneurship scholars, without any traces to follow in our pursuit of understanding them.

To Sum Up, What Do We Know About Serial Entrepreneurship?

The key, therefore, to our investigations of the distinct spaces of firms and entrepreneurs is the phenomenon of serial or multiple entrepreneurs – entrepreneurs who start several firms, some successful and others not. Although several entrepreneurship researchers (MacMillan, 1986; McGrath, 1996; Scott & Rosa, 1996) have urged the necessity to study "habitual" entrepreneurs (i.e., entrepreneurs who enjoy the venture creation process and once established, tend to hand over their ventures to professional managers and go on to start others), very few empirical studies have been conducted and virtually no theoretical development has taken place in this area (Westhead & Wright, 1998). It is clear, however, that serial entrepreneurs form a substantial (a third or more) of new firms in several countries (Birley & Westhead, 1993; Kolvereid & Bullvag, 1993; Ronstadt, 1984; Schollhammer, 1991).

All empirical studies involving serial entrepreneurs (cited above) tend to focus either on the differences between novices and multiple entrepreneurs and/or the effects of experience on the magnitude of firm performance, which so far they have found to be insignificant (Alsos & Kolvereid, 1999). One reason for the lack of significance could be the fact that failed firms are a way for serial entrepreneurs to learn what works and does not work. In other words, if we consider that learning occurs as much through failed startups as through successful ones, learning through serial entrepreneurial experience may not show up as a higher likelihood for the success of any particular firm started by the serial entrepreneur. It will only show up as a higher probability of success for the *entrepreneur* measured over his/her entire career. The proper unit of comparison then would not be novices versus habitual entrepreneurs (because the novice even though failing at his or her first firm may nevertheless go on to succeed as an *entrepreneur* entire entrepreneurship career. For the one time entrepreneur, the firm is an end in itself; whereas for the multiple entrepreneur, the firm is only an instrument toward the achievement of eventual personal "success."

None of the studies so far investigate the role of firms as mortal implements in the entrepreneurs' toolbox as they explore and pursue their own goals, whether such goals may or may not coincide with "objective" measures of firm performance. However, a narrow sliver of evidence from Caves (1998) suggests that at least some new entrants design their firms with early failure in mind, as experiments as it were, to test the waters of potential success in both established and new industries:

To put the point provocatively, we have thought many entrants fail because they start out small, whereas they may start with small commitments when they expect their chances of success to be small. At the same time, small-scale entry commonly provides a real option to invest heavily if early returns are promising. Consistent with this, structural factors long thought to limit entry to an industry now seem more to limit *successful* entry: if incumbents earn rents, it pays the potential entrant to invest for a "close look" at its chances. (1998: 1961)

Hence, the most important difference between the space of firms and the space of entrepreneurs consists in the *stakes* involved in their respective definitions of success and failure. In the case of firms, success/failure is a 0-1 variable. Although there may be firms at the margin whose fate with regard to success or failure may not be perfectly clear, in most cases within a given period of time, firms can be classified either as a success or a failure. In the case of entrepreneurs, however, success usually consists of some proportion of *m* successful firms out of the total of *n* ventures they create. Obviously, in cases where an entrepreneur founds only one firm, *m* and *n* are equal and the space of firms and the space of entrepreneurs become identical. But in cases where an entrepreneur creates more than one firm, the spaces are distinct and for all practical purposes, a serial entrepreneur is considered successful so long as he or she has at least one (substantially) successful founding ($m \ge 1$) over his or her entrepreneurship career. The

implications of this difference in the two spaces are extremely important for a theory of how entrepreneurs can succeed through failure management. We will now develop the theoretical basis to establish this implication, and then trace its consequences to the notion of effectual probability.

ENTREPRENEURS, AS DISTINCT FROM FIRMS: E-SPACE AND F-SPACE

It is not hard to show that if an entrepreneur starts multiple firms, each with a non-zero probability of success, and entrepreneurial success/failure is an aggregate function of firm success/failure, then the probability of entrepreneurial success may be expected to be amplified relative to the probability of firm success. For example, consider an entrepreneur (*e*) who starts *r* firms in sequence, where each firm has a probability *1- q* of success, and the success of any firm is independent of that of another. Define the entrepreneur to be a success if at least one of his *r* firms succeeds. Clearly, the ratio of the probability Pr(e) that the entrepreneur succeeds to the probability that a single firm succeeds, Pr(f) is,

$$amplification = \frac{\Pr(e)}{\Pr(f)} = \frac{1 - q^r}{1 - q}$$
(1)

This model, while the simplest of its kind, nevertheless contains valuable insights on the manipulation of conditioning assumptions to improve probability assessments, the importance of the many-to-one relationship between firms and entrepreneurs, and the concept of probability amplification as an entrepreneurial control mechanism. An formal analysis with a worked out example is given in the Appendix. More detailed analyses with several variations and extensions are available from the authors on request. In the ensuing discussion we will restrict ourselves to key steps in the analysis in order to keep the exposition simple and uncluttered.

Preliminaries

There are two sets: the set of firms (F-space), $F = (f_1, f_2, ..., f_n)$, and the set of entrepreneurs (E-space), $E = (e_1, e_2, ..., e_m)$. The two are related by the following assumptions:

Assumption 1: Every firm in F has exactly one founder in E.

Assumption 2: Every entrepreneur in E is a founder of at least one firm in F.

Then there exists a map Ψ , defined by the rule:

$$\psi : F \rightarrow E,$$
(2)

 $\psi(f_j) = e_i \quad \text{iff } e_i \text{ is the founder of the firm } f_j$

The assumptions do not rule out an entrepreneur in E from starting multiple firms in F. If there are more firms than entrepreneurs (n > m) then Ψ is a many-to-one function. On the other hand, if n = m then Ψ has to be a one-to-one function.

There are two other maps. The *firm success map* $S_f : F \rightarrow (0,1)$ classifies each firm in F as either a success (1) or a failure (0). We are not particular about *how* this assignment is done; what matters is that a firm's fate can be classified into just two groups (success/failure). However, the analogous *entrepreneurial success map* $S_e : E \rightarrow (0,1)$ is dependent (discussed below) on S_f and the association map Ψ . There are several, non-equivalent ways of defining entrepreneurial success, of which the following is perhaps the simplest.

1-of-N Rule: An entrepreneur is classified a success, if at least one of the entrepreneur's firms is classified a success. Mathematically, $S_e(e_i) = 1$ iff there is at least one firm f_j such that $\Psi(f_j) = e_i$.

In what follows, the shorthand " e_i " and " $\overline{e_i}$ " will be used rather than the verbose "the entrepreneur e_i whose $S(e_i) = 1$ and $S(e_i) = 0$ respectively. f_j and $\overline{f_j}$ should also be interpreted in

a similar manner. Finally, when we speak of the firms f_{i_1}, f_{i_2}, K , f_{i_k} created by an entrepreneur e_i , it is understood that the firms are temporally ordered according to their indices, that is, firm f_{i_1} was created before f_{i_2} if and only if $i_1 < i_2$.

The probability of entrepreneurial success

Define the probability space $\Pi = (\Omega^n = \{0,1\}^n, Pr)$, where the sample space Ω^n consists of all possible binary strings (events) from (0,0,...,0) through (1,1,...,1). Each event represents a possible fate for the *n* firms in the set F. For example, (0,0,...,0) represents the event that all the firms in *F* are failures, while (1,0,0,...,0) is the event that the firm f_1 is a success, while the others are all failures and so on. Pr : $\Omega^n \rightarrow [0,1]$ is the joint probability measure associated with the sample space Ω^n . Given an event $b \in \Omega^n$, Pr(b) is the probability of that event happening.

The existence of the mapping $\psi(f_j) = e_i$, implies that the event $\{e_i = 1\}$ is the disjunction of the events $\{f_j = 1\}$ where $\psi(f_j) = e_i$. Hence $\Pr(e_i = 1)$, the probability of success for the entrepreneur e_i , is a function of the probabilities of the events $\{f_j = 1\}$. Of course, there is nothing to prevent the probabilities of the events $\{f_j = 1\}$ from depending on the fates of other firms in F-space. However the following two items should be noted.

First, it is necessary to distinguish between the probability of a *given* entrepreneur succeeding: $Pr(e_i = 1)$, from the probability of *an* entrepreneur succeeding (Pr(e = 1)). The latter probability is given by the formula:

$$\Pr(e=1) = \sum_{i=1}^{m} \Pr(e_i = 1 | e = e_i) \Pr(e = e_i) = \sum_{i=1}^{m} \Pr(e_i = 1) \Pr(e = e_i)$$
(2)

If it is assumed that the probability of picking an entrepreneur e_i is independent of the index *i*, then $Pr(e = e_i) = 1/m$ and the above formula becomes,

$$\Pr(e=1) = \frac{1}{m} \sum_{i=1}^{m} \Pr(e_i = 1)$$
(3)

In entirely the same manner, it can be shown that the probability of a firm succeeding is a weighted average of the probabilities of success of the individual firms and given by,

$$\Pr(f=1) = \frac{1}{n} \sum_{j=1}^{n} \Pr(f_j=1)$$
(4)

Second, $Pr(f_{i_j} = 1)$ or the probability of success for the j^{th} firm created by an entrepreneur *can* depend on the firms created by the entrepreneur in the past, but cannot depend on firms that the entrepreneur will create at a future time. Mathematically, the causality requirement implies:

$$\Pr(f_{i_j} \mid f_{i_k}) = \Pr(f_{i_j}) \qquad \text{if } \psi(f_{i_j}) = \psi(f_{i_k}) \text{ and } i_j > i_k \tag{5}$$

Equation (5) is one reason why the ensemble averages need not be equal to the temporal averages. Still, for establishing our claim that serial entrepreneurship leads to an amplification of the probability of success for a given entrepreneur, the above constraint is not relevant. This is because serial entrepreneurship has two aspects: (a) seriality and (b) multiplicity. It is the multiplicity aspect that is responsible for the amplification effect, though the seriality aspect embodied by Equation (5) may affect the *magnitude* of that amplification effect.

Now, the probability that a firm f_j is successful and its founder e_i is successful is given by,

$$\Pr(f_j, e_i) = \Pr(e_i) \times \Pr(f_j | e_i) = \Pr(e_i | f_j) \times \Pr(f_j)$$
(6)

Equation (6) is nothing more than Bayes formula applied to the event $\{f_i, e_i\}$. From Equation (6)

$$\Pr(e_i) = \frac{\Pr(e_i \mid f_j)}{\Pr(f_j \mid e_i)} \times \Pr(f_j) = \gamma(e_i, f_j) \times \Pr(f_j)$$
(7)

The factor $\gamma(e_i | f_j)$ is defined to be the *Bayes gain* of the event $\{e_i = 1\}$ with respect to the event $\{f_j = 1\}$, or simply, the Bayes gain of e_i with respect to f_j . The Bayes gain of two events is a non-negative real number. The Bayes gain is not defined if the denominator is zero.

The Bayes gain $\gamma(e_i | f_j)$ acts as an amplifying factor on the probability of success for firm f_j . The range of amplification is determined by the range of values $Pr(f_j | e_i)$ can assume, and is given by,

$$\frac{1}{\Pr(f_j)} \ge \gamma(e_i, f_j) = \frac{1}{\Pr(f_j | e_i)} \ge 1$$
(8)

The amplification is smallest (i.e., equals 1) when $Pr(f_j | e_i)$ is maximum. This occurs when an entrepreneur starts one and only one firm. In this case, E-space and F-space are of course isomorphic, and no amplification due to Bayes gain is possible.

On the other hand, the amplification is largest when $Pr(f_j | e_i)$ is minimum. This occurs when the events $\{f_j = 1\}$ and $\{e_i = 1\}$ are independent. The intuitive explanation for this seeming anomaly lies in the fact that as the entrepreneur starts more firms, his or her success (as an *entrepreneur*) depends less and less on the success of any given firm he or she starts. In other words, the Bayes gain increases directly as a function of the entrepreneur's willingness to fail. This implication provides a viable explanation for the phenomenon that Caves (1998) highlighted – viz., that when entrepreneurs believe they are faced with a lower likelihood of success, they tend to invest in smaller experiments with a larger willingness to fail. This also attests to the practical efficacy of the "affordable loss," or in the extreme case the "zero resources to market" principle of the theory of effectuation (Sarasvathy, 2001).

In general, it may be that the relationship of interest is not the relative amplification of the probability of success of a given entrepreneur with respect to a given firm, but the overall relationship between the probability of success for *an* entrepreneur and the probability of success for *a* firm. The general form of the relationship is not particularly illuminating, but if it assumed that all the entrepreneurs create the same number of firms *d* (so that $n = m \times d$) it can be shown that:

$$Pr(e = 1) = \gamma_{avg} \times Pr(f = 1),$$
where, $\gamma_{avg} = \frac{\sum_{j=1}^{n} \gamma(\psi(f_j), f_j) \times Pr(f_j)}{\sum_{j=1}^{n} Pr(f_j)} \ge 1$
(9)

So the overall probability of entrepreneurial success is also amplified relative to the overall probability of firm success. Since every firm f_j is associated with exactly one entrepreneur $\psi(f_j)$, there is a unique Bayes gain factor $\gamma(\psi(f_j), f_j)$, associated with that firm. The weighted average of the factors (with regard to the weights $Pr(f_j)$) is the average Bayes gain factor, γ_{avg} . Rather than detail the relatively straightforward consequences of Equation (9) we shift our attention to further exploring the *meaning* of the notion of Bayes gain.

Serial entrepreneurship as a simple machine: The Bayes Hydraulic Press

The fact that the Bayes gain can be made to be greater than 1 is nothing more than a simple consequence of the fact that the joint probability of the event $\{e_i, f_j\}$ can be evaluated in two different ways (assume $\Psi(f_j) = e_i$ throughout this discussion). Each evaluation corresponds to a particular decomposition of the original event $\{e_i, f_j\}$, namely, either as the event pair ($\{e_i\}$, $\{f_j \mid e_i\}$), or as the event pair ($\{f_j\}$, $\{e_i \mid f_j\}$). Though the probability content of one event pair is the same as that of the other, their semantics are not. Consider the fact that we can very often assign preferences to conditionals. For example, it is perhaps more preferable to know the value of Pr(X has Disease A|X shows symptom S), rather than the value of Pr(X shows symptom S|X has Disease A).

But such semantics and preferences are not part of a probability space's definitions. A probability space looks at the world of events through narrow combinatorial slits, leaving the *decision-maker* with not only the problem of *interpretation* of the significance of a probability, but also the selection of the *means* by which a probability is to be evaluated².

< Insert Figure 1 about here >

A physicist might say that the computation of the joint probability of an event $\{A,B\}$ is *invariant* with respect to its event pairs, $\{A,B|A\}$ and $\{B,A|B\}$. This points to a connection with the logical basis for the construction of simple machines (the pulley, the lever, the wedge, the inclined plane, the screw etc.), integral to the study of statics. The simple machine is predicated

² This is a philosophical position, not a mathematical truth. Consider for example, the position advocated by Jose Ortega y Gasset, the *philosopher of the circumstance*: ``Take any kind of object, apply to it different systems of evaluation, and you will have as many different objects instead of a single one." (*Meditations on Quixote*, translated from the Spanish by Evelyn Rugg and Diego Marin, Note 5: pp. 168).

on the existence of a trade-off between two related parameters of a problem space that can be used to exploit an overall invariance in the solution space. As illustrated in detail in Figure 1, in the case of the hydraulic press, we can choose to move a small piston over a longer distance in order to achieve a smaller movement of a larger piston. The alternative of course is to apply a large force directly to the larger piston to move it where we want it to go.

Similarly, by understanding the trade-off that results in the Bayes gain that we calculated above, entrepreneurs can choose to start several firms with smaller investments and with an acceptance of the fact that some of them might fail. This allows them to amplify the probability of their success irrespective of the probability of success of any given firm they start, while allowing them to control possible losses by keeping the investments lean and mean. The alternative, of course, is to invest larger sums (i.e., whatever it takes) in accurate predictions leading to a single bet on the firm with the highest likelihood of success. This is the familiar course of the optimal strategy, using either unbounded or bounded rationality assumptions. However, given that predictions are notoriously unreliable, especially in the problem space of entrepreneurship characterized mostly by Knightian uncertainty, it is rather comforting that another alternative exists in the form of the Bayesian hydraulic press (Knight, 1921).

It could be argued that serial entrepreneurship is nothing but a diversified portfolio over time, as opposed to concurrent diversification in a normal portfolio. But a little investigation into the features of the two shows almost immediately that the two are vastly different. First, concurrent portfolio diversification requires considerable up-front investments, while serial entrepreneurship can begin with investments as low as zero. Second, while large portfolios may need no predictive strategies in selection of firms, they provide no control to the investor on the overall potential return. The most they can do is reduce risk, given whatever levels of return may be achieved by the individual management teams in each of the firms. Serial entrepreneurship, on the other hand, provides the entrepreneur maximum control possible in effecting potential returns. Third, if it is argued that small portfolios such as those held by venture capitalists do provide some upside control, we then have to deal with the predictive strategies involved in the selection of firms into that portfolio. Serial entrepreneurship, in its turn, allows the entrepreneur to experiment with several ways to put together combinations of stakeholders and to leverage contingencies as they occur to create value without having to invest heavily in prediction (Sarasvathy, 2001). Without belaboring the point further, we can establish that concurrent portfolio diversification is clearly based on predictive rationality, while serial entrepreneurship need not be. In a sense, these two approaches to managing uncertainty are non-ergodic – i.e., temporal averages are not equivalent to ensemble averages.

At first glance, the Bayesian hydraulic press, aside from illustrating the trade-off between the willingness to fail and the pitfalls of prediction appears to provide no additional illumination. But further investigation shows that the work it does in our conceptualization is of considerable philosophical and pragmatic import, since it allows us to grasp first hand how sensitive probability assessments are to their conditioning assumptions. The import lies in the realization that: to the extent that the conditional assumptions are not set in stone, but may be modified through human action (specifically by the action of the entrepreneur in our case), modeling the probability assessment as a simple machine reveals the particular conditioning assumptions to be invalidated by entrepreneurial action.

Bayes' formula has traditionally been used as an <u>inference engine</u> – i.e., to update our beliefs in the face of states of the world actually realized. Modeling it as a simple machine reveals that it is capable of another use, namely, as a <u>control engine</u> – i.e. it can be used to

manipulate states of the world (to the extent that the assumptions it is conditioned on are manipulable) to align with our beliefs. Thus what the conditioning assumptions are, how we choose them, and to what extent and in what ways we can manipulate them all become extremely relevant issues in the formulation of the problem we present in this paper.

To return to the concrete case of serial entrepreneurship, the Bayesian hydraulic press sharply highlights the fact that probabilities in F-space need not be used to merely update probability assessments in E-space, instead they can be used to *control* event probabilities in Espace. In the trivial case, the one we have thus far examined, we can interpret this to mean starting more than one firm. The real pay-off, however, awaits us in putting the Bayesian hydraulic press to work to move Mount Improbable.

Moving Mount Improbable: Toward a theory of effectual probability

The Bayes gain in the case of the serial entrepreneur can be interpreted in two ways. In the first, the entrepreneur reasons as follows: *I observe that the probability of firm failure is very high; therefore I will start several firms*. This is the normal way of interpreting Bayes' rule -- as an inference engine. In the simple machine interpretation, however, the entrepreneur reasons as follows: *Irrespective of what the probability of firm failure is, I can increase the probability of "my" success in the following ways – one of the ways being serial entrepreneurship*. Although both interpretations result in serial entrepreneurship, there is a fundamentally different approach to the decision in each of the two interpretations. And this difference in approach leads to crucial differences in the way the decision maker perceives, formulates, and executes possible strategies that operationalize the decision.

In the conventional interpretation of the Bayes gain, all events are fully funded in their probabilities. In other words they are analogous to the probability of rain – while knowing the probability allows us to take action (carry an umbrella, stay home etc.) to prevent its consequences (getting wet), the probability of the event itself is given, and cannot really be changed. In the simple machine interpretation of the Bayes gain, however, not all events are fully funded. Instead, events are divided into three categories according to how controllable they are through human action. In general, (a) some events may be fully funded and beyond the decision maker's control; (b) others may be free or fully within the control of the decision maker; and (c) yet others may be as yet unfunded or controllable to some extent and under certain circumstances. Obviously, in the case involving events of type (a), Bayesianism can only be used as an inference engine. In cases involving events of types (b) and (c), however, not only can the Bayesian hydraulic press be used to increase or decrease their probabilities, but it can also be used to help identify specific strategies of how to do so. In other words, in the cases of free and unfunded events, the decision may be driven by the logic of control rather than the logic of prediction. And the decision maker can use effectual as well as causal strategies and processes (Sarasvathy, 2001).

A more detailed theory of effectual probability would be beyond the scope of this paper and is being developed as a separate thesis in Menon and Sarasvathy (2002).

SUMMARY AND POSSIBILITIES FOR FUTURE WORK

In this paper we set out to investigate if we can say anything more about entrepreneurial success and failure than the oft-repeated, well-accepted, and pragmatically bankrupt bromide, *"Most firms fail."* A careful exploration of the empirical work to date on this issue revealed the

existence of two distinct probability spaces -- the space of firms (F-space) and the space of entrepreneurs (E-space) -- and the fact that the two were often confounded in the designs of the studies. This confounding ended up clouding the results of the studies and made interpretations of the results either irrelevant or unusable.

Delving deeper into the two spaces and the relationship between the two led us to the following three key findings:

- Probabilities defined over E-space may assume different values than probabilities defined over F-space. Accordingly, decision making in E-space is not necessarily identical with decision making in F-space.
- 2. Bayes' formula of conditional probabilities can be interpreted as a simple machine to *control* event probabilities in E-space, rather than as an inference engine that merely *updates* probabilities in E-space in the face of new evidence in F-space.
- 3. Firm successes and failures do not *determine* the successes and failures of entrepreneurs. In fact, entrepreneurs can *use firms as instruments* to increase the probabilities of their own success.

The last finding has larger implications for entrepreneurial learning that have to be investigated and developed through future work. In fact, in the interests of an uncluttered exposition of a new conceptualization of entrepreneurial success and failure, we have altogether ignored the treatment of learning effects of our model in this paper. Although significant positive effects of learning can add to the Bayes gain to increase the probability of success for the entrepreneur, our entire discussion has been centered on the problem of making the Bayesian hydraulic press work, *without taking into account the learning effects* involved in serial entrepreneurship. But clearly this has to form a vital area of inquiry into the phenomenon of serial entrepreneurship and in the further development of our model of it as a simple machine.

CONCLUSION

In a poem titled "What this mode of motion said" the poet A. R. Ammons (1971) speaks in the voice of a protagonist who describes himself as "I am the wings when you me fly." We might as well think of the voice as the voice of Bayes' formula speaking to us:

> pressed for certainty I harden to a stone, lie unimaginable in meaning at your feet,

leave you less certainty than you brought, leave you to create the stone as any image of yourself, shape of your dreams:

This poem suggests a way for entrepreneurship scholars to pick up the gauntlet that the great economist Arrow threw down at the beginning of this paper. Perhaps the surest way to falsify his null hypothesis -- that there is no particular set of individual or institutional characteristics that separate the failures and successes -- is to accept it. This is not a paradox. We only need to understand that the null hypothesis does not exclude the possibility that *all* entrepreneurial individuals and institutions can succeed, *irrespective of the null hypothesis being true for firms*, provided they choose to pick up the stone of Bayesian uncertainty and start carving the futures they imagine possible, smoothing their way through the ragged edges of firm failures.

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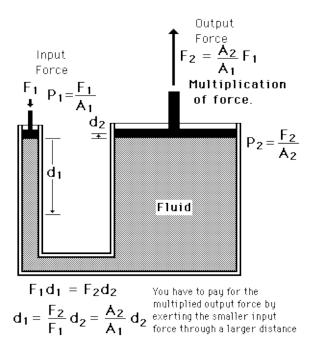
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Figure 1

Simple machines are based on three principles: The first is the existence of an invariant quantity, like Work or Energy (It should be kept in mind that the invariance may only be with respect to certain idealized situations, and not backed by a general conservation principle). Second, these invariant quantities are expressible in terms of non-invariant quantities; for example, Work(W) = Force(F) times Distance(d) or Pressure(P) = Force(F) divided by Area(A). Finally, there should be a preference ordering on the non-invariant quantities, such that we are willing to tradeoff a preferred change in one quantity for a change in the other.



A good example of a simple machine is the hydraulic press which is based on Pascal's principle which asserts that pressure is transmitted without modification in an incompressible fluid. The figure above shows the principle in action. A small force exerted on the piston at one end is multiplied many times at the other; the amplification factor is given by the ratio of the areas of the two piston surfaces. This amplification factor is called *the mechanical advantage* of the simple machine. With respect to Pascal's principle, the tradeoff is that the larger force is distributed over a larger area. The hydraulic press can also be explained in terms of the conservation of work as well. With respect to the conservation of work principle, the tradeoff is that the smaller force has to be applied over a larger distance.

Analogous to the invariance principle of a simple machine, The Bayes rule asserts the invariance of a joint probability of a pair of random variables under two different evaluations.

Appendix

1.1 Bayes Gain

Define the probability space $\Pi = (\Omega^n = \{0,1\}^n, Pr)$, where the sample space Ω^n consists of all possible binary strings (events) from (0,0,...,0) through (1,1,...,1). Each event represents a possible fate for the *n* firms in the set *F*. For example, (0,0,...,0) represents the event that all the firms in *F* are failures, while (1,0,0,...,0) is the event that the firm f_i is a success, while the others are all failures and so on. Pr : $\Omega^n \rightarrow [0,1]$ is the joint probability measure associated with the sample space Ω^n . Given an event $b \in \Omega^n$, Pr(b) is the probability of that event happening. Also, since $S(e_i)$ is dependent on the values of $S(f_j)$ where $\Psi(f_j) = e_{i,j}$ the probability $S(e_i) = 1$ (or 0) can be defined in terms of the probability measure Pr.

The probability that a firm f_i is successful and entrepreneur e_i is successful is given by,

$$\Pr(f_j, e_i) = \Pr(e_i) \times \Pr(f_j | e_i) = \Pr(e_i | f_j) \times \Pr(f_j)$$
(1)

Equation (1) is nothing more than Bayes Formula applied to the event $\{f_i, e_i\}$. From Equation (1),

$$\Pr(e_i) = \frac{\Pr(e_i \mid f_j)}{\Pr(f_j \mid e_i)} \times \Pr(f_j) = \gamma(e_i, f_j) \times \Pr(f_j)$$
(2)

The factor $\gamma(e_i, f_j)$ is defined to be the *Bayes gain* of the event $\{e_i = 1\}$ with respect to the event $\{f_j = 1\}$, or simply, the Bayes gain of e_i with respect to f_j . The Bayes gain of two events is a non-negative real number. The Bayes gain is not defined if the denominator is zero.

Assume that the probability of the success of the j^{th} firm in F is not identically zero, that is,

$$\Pr(f_i) > 0 \quad \forall f_i \in F \tag{3}$$

The Bayes gain can be calculated with respect to cases two of interest: (s) e_i is the founder of f_j , and (b) e_i is not the founder of f_j . Each case constitutes an additional piece of information that needs to be listed in the conditional (for example, $Pr(f_j | e_i, \Psi(f_j) = e_i)$ rather than $Pr(f_j | e_i)$, but to avoid notational clutter, we shall simply assume that the context will make it clear which case is under discussion.

Case 1 ($\Psi(f_i) = e_i$): From Equation (2) above,

$$\Pr(e_i) = \frac{\Pr(e_i \mid f_j)}{\Pr(f_j \mid e_i)} \times \Pr(f_j) = \gamma(e_i, f_j) \times \Pr(f_j)$$
(4)

Since an entrepreneur has been defined as being successful provided at least one of the firms started by the entrepreneur is a success, the numerator conditional probability evaluates to:

$$\Pr(e_i \mid f_j) = 1 \tag{5}$$

The moment it is given that a firm f_j started by an entrepreneur e_i is a success, there is no more uncertainty regarding the success of that entrepreneur. Note that Equation (4) is equivalent to assuming that,

$$\Pr(\overline{e}_i | f_j) = 0 \implies \Pr(f_j | e_i) = 0 \implies \Pr(f_j | \overline{e}_i) = 1$$
(6)

Substituting Equation (4) in Equation (2),

$$\Pr(e_i) = \frac{1}{\Pr(f_j \mid e_i)} \times \Pr(f_j) = \gamma(e_i, f_j) \times \Pr(f_j)$$
(7)

Since $Pr(f_j) > 0$ by Equation (3) and $Pr(e_i | f_j) = 1$ by Equation (4), it can be seen from Bayes formula that, $1 \ge Pr(f_j | e_i) > 0$. Hence,

$$\gamma(e_i, f_i) \ge 1 \tag{8}$$

On the other hand, since $Pr(e_i) \le 1$, from Equation (6) it must always be the case that,

$$\gamma(e_i, f_j) \le \frac{1}{\Pr(f_j)} \tag{9}$$

Accordingly,

$$\Pr(e_i) = \frac{\Pr(f_j)}{(\Pr(f_j|e_i))} \ge \Pr(f_j)$$
(10)

Thus the Bayes gain $\gamma(e_i, f_j)$ acts as an amplifying factor on the probability of success for firm f_j . The range of amplification is determined by the range of values $Pr(f_j|e_i)$ can assume, and from inequalities (7) and (8) it follows that,

$$\frac{1}{\Pr(f_j \mid e_i)} \ge \gamma(e_i, f_j) = \frac{1}{\Pr(f_j \mid e_i)} \ge 1$$
(11)

The amplification is smallest when $Pr(f_j | e_i)$ is maximum. This occurs when $\psi(.)$ is a one-to-one map; every entrepreneur starts one and only one firm. In this extreme case, if e_i is known to be a success, and f_j = $\psi(e_i)$ then it has to be the case that f_j is a success as well (from the definition of entrepreneurial success). Hence, when $\psi(.)$ is a one-to-one function,

$$\Pr(f_j | e_i) = \Pr(e_i | f_j) = 1$$
(12)

Consequently the Bayes gain is 1, and from Equation (6),

$$\Pr(e_i) = \Pr(f_i) \tag{13}$$

Intuitively, when *E*-space and *F*-space are isomorphic, the same measure governs firm success as well as entrepreneurial success.

On the other hand, the probability amplification is largest when $Pr(f_j | e_i)$ is minimum. This occurs when the events $\{f_j = 1\}$ and $\{e_i = 1\}$ are independent. Consequently $Pr(f_j | e_i) = Pr(f_j)$. From Equation (6),

$$\Pr(e_i) = \gamma(e_i, f_j) \times \Pr(f_j) = \frac{\Pr(f_j)}{\Pr(f_j | e_i)} = \frac{\Pr(f_j)}{\Pr(f_j)} = 1$$
(14)

Thus, when the Bayes gain is at a maximum, the success of e_i is guaranteed.

Case 2 $(\Psi(f_j) \neq e_l)$: In the case e_i is not the founder of firm f_j (as before, we ignore listing this information in the conditionals),

$$\Pr(e_i) = \frac{\Pr(e_i \mid f_j)}{\Pr(f_j \mid e_i)} \times \Pr(f_j) = \gamma(e_i, f_j) \times \Pr(f_j)$$
(15)

It is reasonable to assume that,

$$\Pr(f_j | e_i) = \Pr(f_j) \tag{16}$$

$$\Pr(e_i | f_j) = \Pr(e_i) \tag{17}$$

In other words, the events $\{e_i = 1\}$ and $\{f_j = 1\}$ may be taken to be independent *if* e_i is not the founder of the firm f_j . Consequently,

$$\gamma(e_i, f_j) = \frac{\Pr(e_i \mid f_j)}{\Pr(f_j \mid e_i)} = \frac{\Pr(e)}{\Pr(f_j)}$$
(18)

In this case, nothing can be said about the Bayes gain $\gamma(e_i, f_j)$ It can be less, equal to or greater than 1.

1.2 Surviving failure

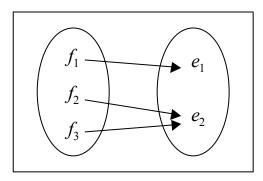
One quantity of great interest is, $Pr(e_i = 1 | f_j = 0)$, the probability of the entrepreneur e_i surviving the failure of one of his/her firms f_j . It will now be shown that this probability is closely related to the Bayes gain.

$$\begin{aligned} \Pr(f_{j} - 0) &\times \Pr(e_{i} = 1) | f_{j} = 0 \rangle &= \Pr(f_{j} = 0 | e_{i} = 1) \times \Pr(e_{i} = 1) \\ \Rightarrow & \Pr(e_{i} = 1) | f_{j} = 0 \rangle = (1 - \Pr(f_{j} = 1 | e_{i} = 1)) \frac{\Pr(e_{i} = 1)}{\Pr(f_{j} = 0)}, \\ &= (1 - \frac{1}{\gamma(e_{i}, f_{j})}) \frac{\Pr(e_{i} = 1)}{\Pr(f_{j} = 1)} \times \frac{\Pr(f_{j} = 1)}{\Pr(f_{j} = 0)}, \end{aligned}$$
(19)
$$&= (1 - \frac{1}{\gamma(e_{i}, f_{j})}) \gamma(e_{i}, f_{j}) \times \frac{\Pr(f_{j} = 1)}{\Pr(f_{j} = 0)}, \\ &= (\gamma(e_{i}, f_{j}) - 1) \times \frac{\Pr(f_{j} = 1)}{\Pr(f_{j} = 0)} \end{aligned}$$

Assuming the probabilities of firm failure and firm success are fully funded (i.e., they are outside the control of the entrepreneur -- externally determined by the environment, for example), it follows that the probability of the entrepreneur e_i surviving the failure of one of his/her firms f_j is directly proportional to the entrepreneur's Bayes gain with respect to that firm.

1.3 A worked-out example

Suppose e_1 and e_2 are two entrepreneurs. Suppose e_1 starts one firm f_1 , and e_2 starts two firms, f_2 and f_3 . Then the economy would like the following:



In this economy, there are eight possible events; each event corresponds to a possible scenario for the firms. Each event induces an event for the associated entrepreneurs (the entrepreneurs succeed/fail). The whole situation can be represented in the following table.

f ₁	f 2	f ₃	e1	e ₂
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

Assuming all the scenarios are equally likely, what can we say about the probabilities of success of these two entrepreneurs?

$$Pr(e_1 = 1) = \frac{\# \text{ of scenarios in which } e_1 = 1}{\text{Total } \# \text{ of scenarios}} = \frac{4}{8} = 0.5$$
(1)

$$Pr(e_2 = 1) = \frac{\# \text{ of scenarios in which } e_2 = 1}{\text{Total } \# \text{ of scenarios}} = \frac{6}{8} = 0.75$$
(2)

The increase in $Pr(e_2)$ is because e_2 has started two firms.

What about the probability of an entrepreneur succeeding? There are two ways to calculate this.

Method I: For each scenario, we can calculate the probability that an entrepreneur succeeds, and then use Bayes formula as:

$$Pr(e = 1) = Pr(e = 1 | \{f_1, f_2, f_3 = 0, 0, 0\}) \times Pr(f_1, f_2, f_3 = 0, 0, 0) + Pr(e = 1 | \{f_1, f_2, f_3 = 0, 0, 1\}) \times Pr(f_1, f_2, f_3 = 0, 0, 1) + \dots + Pr(e = 1 | \{f_1, f_2, f_3 = 1, 1, 1\}) \times Pr(f_1, f_2, f_3 = 1, 1, 1)$$
(3)

The conditional probabilities of $Pr(e = 1 | f_1, f_2, f_3 = 0, 0, 0)$ are listed in the following table:

f ₁	f ₂	f_3	<i>Pr(e</i> <i>f</i> ₁ , <i>f</i> ₂ , <i>f</i> ₃)
0	0	0	0
0	0	1	1/2
0	1	0	1/2
0	1	1	1/2
1	0	0	1/2
1	0	1	1
1	1	0	1
1	1	1	1

For example, when $f_1, f_2, f_3 = 1, 0, 0, e_1 = 1$ but $e_2 = 0$. So there is a 50\% chance that an entrepreneur in this scenario will be successful.

Since we have assumed all the 8 scenarios are equally likely (i.e. probability = 1/8), we get for the overall probability of success,

$$\Pr(e=1) = \frac{1}{8} (0+0.5+0.5+0.5+0.5+1+1+1) = \frac{5}{8}$$
(4)

Method II: This is the method we used in the paper:

$$Pr(e=1) = Pr(e=e_1)Pr(e_1=1|e=e_2) + Pr(e=e_2)Pr(e_2=1|e=e_2)$$
(5)

Assuming we can pick either entrepreneur with equal probability (that is, $Pr(e = e_1) = Pr(e = e_2) = 0.5$), we get,

$$Pr(e=1) = 0.5 \times Pr(e_1=1) + 0.5 \times Pr(e_2=1) = 0.5 \times \frac{1}{2} + 0.5 \times \frac{3}{4} = \frac{5}{8}$$
(6)

Next, what about the probability that a firm succeeds?

It is the average of the probabilities of the 3 firms succeeding. That is, from Bayes formula:

$$Pr(f = 1) = Pr(f = f_1)Pr(f_1 = 1 | f = f_2) + Pr(f = f_2)Pr(f_2 = 1 | f = f_2) + Pr(f = f_3)Pr(f_3 = 1 | f = f_3)$$
(7)

This implies,

$$\Pr(f=1) = \frac{1}{3} \{ \Pr(f_1=1) + \Pr(f_2=1) + \Pr(f_3=1) \}$$
(8)

and hence,

$$\Pr(f=1) = \frac{1}{3}(0.5 + 0.5 + 0.5) = 0.5$$
(9)

Finally, what about the overall probability of the entrepreneur over that of the firm?

Notice that Pr(e) = 5/8 but Pr(f) = 0.5. So now we can calculate the overall probability of entrepreneurial success over firm success, or the Bayes gain in this toy economy as follows:

$$\gamma_{overall} = \frac{\Pr(e=1)}{\Pr(f=1)} = \frac{5/8}{1/2} = \frac{10}{8} > 1$$
 (10)