EMERGING FIRMS AND THE ALLOCATION OF CONTROL RIGHTS: A BAYESIAN APPROACH

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This paper suggests that founders often use firm formation to exploit opportunities and must sometimes make organizing decisions about the allocation of control before the economic value of the opportunity can reliably be known even probabilistically. Motivated by questions surroundings such settings, we use incomplete contract theory and apply a Bayesian learning model to the allocation process of ownership control rights of founders in emerging firms. This model examines how founders learn and build on their prior beliefs, enabling them to allocate and change ownership control rights under differing conditions of risk and uncertainty.

Incomplete contract theory (ICT) of the firm focuses on the governance of exchanges characterized by relationship-specific investments whose future outcomes cannot be explicitly contracted for at the time a firm is organized to exploit these exchanges (Grossman & Hart, 1986). This theory suggests that these exchanges are plagued by numerous unforeseen and, thus, unverifiable contingencies (Grossman & Hart, 1986; Hart & Moore, 1988). By definition, these exchanges cannot be included in a complete contingent claims contract (Williamson, 1975). Rather than attempting to specify these contracts as complete ex ante, ICT suggests that parties in this type of exchange will specify those contingencies that can be specified and will assign residual decisions rights—what the theory calls "residual rights of control"-to one party or the other. ICT also suggests which party to an exchange should receive these residual rights in order to maximize its overall economic value: the party that expects to gain the most from this exchange (Grossman & Hart, 1986). This is because this party has the strongest incentives to maximize the total value of an exchange.

However, what if, at the time a firm is organized, the party with the most to gain from an exchange is not known? In this setting, to whom are residual rights of control assigned, and how is the firm organized?¹ These are important questions, because founders often make decisions about residual rights of control in emerging firms before reliable information about the set of outcomes is available and the probability distribution associated with specific investments in a market opportunity is known (Alvarez & Barney, 2005; Foss, Foss, & Klein, 2007; Foss & Klein, 2005). How do founders in these settings make these residual rights of control decisions, especially when two or more individuals are involved in the creation of a firm? Moreover, the lack of information and unreliability about the set of outcomes and the probability distribution associated with specific investments may result in the wrong allocation of residual control rights, requiring changes to ownership as information is updated (Bruton, Fried, & Hisrich, 1997; Lerner, 1995). What long-term consequences do these early organizing decisions have on the structure and performance of a firm? The purpose of this paper is to explore some answers to these questions.

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¹ There is a tension between the argument that the ICT framework relies on unanticipated contingencies and therefore a deep kind of uncertainty, even ignorance, and the argument that it is consistent with a risk-based framework. This argument has been at the heart of much debate within the last decade in contract theory. See, for example, Buckley and Michie (1996) and Foss (2000).

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Cyert and DeGroot (1987) suggest that Bayesian learning models may help scholars explore decision making under conditions of uncertainty. Using Bayesian learning models, here we examine how the prior beliefs of entrepreneurs can cause important differences in ownership structure early in the evolution of an entrepreneurial firm. Bayesian models recognize that decision makers often have imperfect information concerning some important aspect of their decision-making settings, about which they form (possibly incorrect) subjective beliefs. In this approach agents are said to be subjectively rational when they behave in ways that are consistent both with what is known about their decision-making setting and with their beliefs about that setting. This does not suggest that agents have correct beliefs about the setting in which they are making decisions, nor does it suggest that their beliefs will be correct after incorporating new information. The assumption of subjective rationality coupled with Bayesian learning makes it possible to examine the implications of how agents update their beliefs, regardless of what those beliefs were initially. These models explain how decisions are made on the basis of these prior beliefs, how beliefs are updated to incorporate new information, and how decisions are modified when beliefs change.

We begin the article by outlining a basic model of the productivity of a new firm started by two founders. We then explain Bayesian learning within the context of a model of the founders' data and beliefs, which evolve as the founders learn. The section that follows defines conditions of "risk" and "uncertainty" and derives results about the evolving firm structure and allocation of ownership rights under each of these conditions. A common feature of new ventures is that they are often started by more than one founder and the initial organizational structure of shared control is often replaced as one founder emerges as the sole controller of the venture (Kauffman Center for Entrepreneurial Leadership, 1999). Another feature of new ventures is that control can shift several times between the founders.²

In the final section we discuss implications of this research for the field of entrepreneurship, along with some specific directions for future research. We conclude with a discussion about the potential wider applicability of Bayesian modeling in management and organizational research more generally.

A BASIC PRODUCTIVITY MODEL

The model begins with the two founders, A and B, each investing their time and human capital in a new venture.³ During the formation process, each founder experiences a separate event in each period, which both helps the founder learn about the likelihood of future events and shapes the founder's productivity.

We define an event here as experiential knowledge obtained from trading and interacting in the marketplace. Events are interactions with the environment from which founders are able to obtain information, know-how, and feedback for their tasks. Such events are potential sources of learning as each founder experiences market feedback about the product, service, competitors, industry, market, or customers (Argote, 1999; Argote, Beckman, & Epple, 1990). Each founder obtains one event datum per period. Both founders start at time 0. Let $t = 1, 2, \ldots$ denote the integer number of discrete subsequent periods during the founding process. A's datum at time t is denoted by x_t , and B's is denoted by y_t . These data are random draws from a common distribution. This construct reflects the stochastic nature of the market environment in which the new firm is emerging.⁴

The Value of Events

The founders individually experience one event during each and every time period. These

² An example is Sun Microsystems, which was founded by three entrepreneurs; within three years, one of the entrepreneurs, Vinod Khosla, had left the firm. And consider Wil Shipley, one of the founders of The Omni Group, who left after numerous conflicts with the other founders. Shipley then cofounded and launched a new Seattle-based firm,

Delicious Monster, with cofounder Mike Matas. Matas quit after just one year, leaving Shipley as the sole controller.

³ All of the results in this paper generalize naturally to more than two founders. We stick with the case of two founders for expositional simplicity.

⁴ A common distribution can be justified on the grounds that the founders are located in the same business and so face random realizations drawn from a common environment. However, because even in a common environment not everyone experiences exactly the same thing at exactly the same time, founders receive different specific draws in any period.

events confer two benefits. Regardless of which founder experiences the event, that event enables both founders to learn and update their prior beliefs. However, the founder who experiences the event reaps a productivity gain by virtue of the experience itself (Argote, 1999; Cyert & March, 1992; Huber, 1991). While the other founder can freely observe the value of the event and benefits from the event by updating prior beliefs (Kalnins, Swaminathan, & Mitchell, 2006), this founder will not reap the productivity benefits because the founder did not personally experience the event (Dosi, Nelson, & Winter, 2000).

Thus, if founder A experiences x_t and founder B experiences y_t at time t, both A and B will update their prior beliefs with the same information (x_t, y_t) , while only A will gain productively from x_t and only B will gain productively from y_t .

Denote A's productive capability at time t by $\theta_A(t)$, and denote B's by $\theta_B(t)$. Initial abilities are denoted by $\theta_A(0)$ and $\theta_B(0)$, respectively. We assume that early experiences shape founders' abilities more than later experiences. For example, in the early stages of firm formation, founders may make sequential investments of relation-specific human capital, which are costly to reverse and which also affect the productivity of later investments (Mosakowski, 1997; Wadeson, 2004). The importance of these early costly to reverse investments can be represented by a weighting scheme $\{\varphi_t\}$, where $\varphi_{t+1} \leq \varphi_t \forall_t; \varphi_1 = 1$. Then individual founder productivity develops according to⁵

$$\theta_A(t) = \sum_{\tau=1}^t \varphi_\tau \mathbf{x}_\tau, \ \theta_B(t) = \sum_{\tau=1}^t \varphi_\tau \mathbf{y}_\tau.$$
(1)

Although individual productivity and the productivity of the firm are distinct, the productivity of the founders affects the productivity of the emerging firm (Haas, 2006; Ucbasaran, Lockett, Wright, & Westhead, 2003; Zahra, Ireland, & Hitt, 2000). In the next section we explain how individual productivity influences the productivity of the firm.

The Productivity of the Firm

Here we assume that during the early-stage formation of the firm, the sequencing of events is as follows. At each point in time t, the firm structure can potentially change, and this change will determine how the payoffs to the two founders will be distributed in the next period, t + 1 (Bereby-Meyer & Roth, 2006; Kalnins et al., 2006). Once returns arrive at t + 1, the firm structure can once again be changed, and the effects of this change will take place in t + 2. Thus, firm structure is not fixed but can potentially change in each period.⁶

The firm can be organized in two ways: either under shared control or with a single controller. If control is shared, firm productivity is the average productivity of both founders, and both founders are residual claimants.⁷ The productivity of the firm at time t is therefore the random quantity $\overline{\theta}(t) = \frac{1}{2}(\theta_A(t) + \theta_B(t))$. We assume that this is shared equally by the two founders. Of course, an alternative contract could be proposed, making the shares dependent on the individual productivities of the founders. However, this contract could be costly to implement if individuals have incentives to inflate claims about their own contribution, entailing costly monitoring and negotiation. For simplicity, we will assume these costs are too large for either party to agree to an unequal-share statecontingent splitting rule. This also reflects realworld contracts where (at least in cases where initial investments are similar) equal splitting under shared control is the norm.

If control is not shared, we assume that the firm's total product depends only on the controller's individual productivity—that is, is given by

⁵Note that ability evolves stochastically; for simplicity, and to make our main points most clearly, investments of relation-specific human capital are not endogenized as choice variables.

⁶ We implicitly assume that both founders are needed at the outset. Hence, both founders are complementary for the founding event itself (in the sense that both do best by forming one team of two rather than two separate teams). But they are not necessarily complementary thereafter. This could be, for example, because two founders are needed to acquire resources and gain legitimacy, but once those are in place, control structures can take on a variety of forms and trajectories, as we go on to demonstrate.

⁷ While it would be straightforward to replace this setup with a weighted average, it would add nothing to the analysis, apart from introducing an arbitrary distinction between the two founders. An alternative possibility is that there are synergies between founders' abilities that can be realized under shared control (Alchian & Demsetz, 1972). We do not consider this possibility in the paper.

 $\theta_A(t)$ if A is the controller or $\theta_B(t)$ if B is the controller and the controller is the residual claimant. The reason is that once an individual has sole control, he or she is able to leverage what he or she has learned about his or her specific investments so as to maximize the productivity of the firm and, thus, create the most value for him/herself.

The Allocation of Ownership Rights

If control is shared at time *t*, both A and B will receive $\frac{1}{2}\overline{\theta}(t)$. However, as each founder experiences an event in time *t*, his or her expectations of potential returns at t + 1 may differ. In period *t*, E_{jt} denotes *j*'s expectations about the next period t + 1, where *j* is either A or B and *j*' is the other. For example, *j*'s expected return under shared control at t + 1 is $\frac{1}{2}E_{jt}[\overline{\theta}(t + 1)]$, while the corresponding expected return for *j*' is $\frac{1}{2}E_{i't}[\overline{\theta}(t + 1)]$.

In cases where control is not shared, the controller is the residual claimant of ownership rights, while any noncontroller takes an agreed salary or wage. The residual claimant keeps the difference between the (random) realized total product of the firm and the other founder's salary. This could be either positive or negative, and in the case of a loss, the residual claimant absorbs this loss. Both founders are assumed to obtain an ongoing exogenous private benefit, worth $\chi \ge 0$ in monetary terms, from the very act of exercising control (Alvarez & Barney, 2005). One can think, for example, of χ as a "vanity bonus," which is compensation for surrendered power and status that a founder requires in return for selling his or her control rights to the other founder. The vanity bonus helps explain why a founder granted residual rights of control may be unwilling to relinquish those rights, even when it becomes apparent that someone else in the exchange should have residual rights of control and when relinquishing them would increase the value of the exchange.

The firm formation process is a complex situation where the available information and the payoffs tend to be noisy (Bereby-Meyer & Roth, 2006). Differences in information draws cause founders to have different expectations of what they will receive in the next period under both sole control and shared control. Control can change from shared to sole if it is in both founders' interests and if both agree about the benefits of changing control. Founder j' ceding control would require compensation of at least the amount he or she would expect to receive under shared control, plus compensation for loss of control. This is represented by $[\frac{1}{2}E_{j't}(\bar{\theta}(t + 1)) + \chi]$, where $\frac{1}{2}E_{j't}(\bar{\theta}(t + 1))$ is j''s expected return under shared control and χ is j''s required compensation for loss of control.

Founder *j* taking control would require an expected return under sole control that was greater than what *j* would expect to receive under shared control. This founder's expected return under sole control is $E_{jt}[\theta_j(t+1)] - [\frac{1}{2}E_{j't}(\bar{\theta}(t+1)) + \chi]$, which is the difference between two terms. The first term $E_{jt}[\theta_j(t+1)]$ is *j*'s gross expected return under sole control, and $[\frac{1}{2}E_{j't}(\bar{\theta}(t+1)) + \chi]$ is the required compensation that must be paid to *j*' as explained above. The difference between these terms is *j*'s expected return of founder *j* under shared control is $\frac{1}{2}E_{it}[\bar{\theta}(t+1)]$.

Hence, for founder *j* to be willing to move from shared to sole control, the following condition must hold:

$$E_{jt}[\theta_{j}(t+1)] - [\frac{1}{2}E_{j't}(\bar{\theta}(t+1)) + \chi]$$

> $\frac{1}{2}E_{jt}[\bar{\theta}(t+1)].$ (2)

If (2) does not hold for j = A, then A will not become the sole controller. (If it does hold, A will become the sole controller.) Likewise, if (2) does not hold for j = B, then B will not become the sole controller either. (If it does hold, B will become the sole controller.) If (2) holds for neither founder, shared control is agreed at t.

Note that Equation 2 merely states the condition for a change of control structure to create surplus that is valuable to both founders, so leading to this change of structure. While we have supposed that the surplus goes wholly to the new sole controller, it is quite possible for the surplus to be shared in a variety of ways according to the relative bargaining power of the founders. But whether or how the surplus is shared is not of direct interest in this paper, where our focus is purely on changes in the allocation of control rights and its implications for performance. Because the issue of how surplus is shared lies outside the scope of this paper, we do not pursue it further.

The performance of the firm is maximized when the ablest founder has sole control (Reagans, Argote, & Brooks, 2005). The performance of the firm will not be maximized if the least able founder has sole control or if abilities are different and the firm is run under shared control. Shared control can occur when founders' abilities are different if the vanity bonus, χ , is sufficiently high that Equation 2 does not hold for either founder. If the performance of the firm is maximized, this should be associated with greater survival prospects for the firm. Hence, ownership structure is important for understanding how emerging firms evolve and also as an explanation for their subsequent survival.

BAYESIAN MODEL OF LEARNING

In Bayesian decision theory, individuals are assumed to observe new data at time t to update their prior beliefs that were held at t - 1, in order to obtain better-informed posterior beliefs at t(Lee, 1995). Updating takes place according to Bayes' Rule, which, for founder A, for example, states that

$$p_{At}(\mu|\mathbf{x}) \propto p_A(\mu) p(\mathbf{x}_t|\mu). \tag{3}$$

Here $p_{At}(\mu|\mathbf{x})$ is a probability distribution describing A's posterior belief at *t* about some unknown parameter μ conditional on data *x*. This is proportional to (α) the product of A's prior beliefs about μ (namely, $p_A(\mu)$) and the probability of observing data \mathbf{x}_t given μ (namely, $p(\mathbf{x}_t|\mu)$). An analogous version of (3) also holds for B, in which *B* replaces *A* and *y* replaces *x*.

As founders update their beliefs, they realize that they are learning about the environment in which firm formation is occurring. The process of learning about their environment enables the founders to further understand the distribution of possible outcomes (McGrath, 1997, 1999). It is important to note that Bayesian updating assumes that individuals make full and rational use of available information and can assess probabilities in a way that makes Equation 3 operational. To be precise, founders are assumed to be subjectively rational, which is a weaker condition than objective rationality, which requires founders to know the true environmental process driving outcomes (Ryall, 2003: 939). In subjective rationality founders' theories about their environment are consistent with available information. Their actions, which are consistent with their theories about the environment, may, in turn, influence the consequences they ultimately experience. This implies a direct connection between founder behavior and subjective rationality. Subjective rationality includes a behavioral component in that the consequences of a founder's experiences are in part a result of the founder's actions. These interrelationships form a closed feedback loop: founders take actions that are consistent with their beliefs, but what they believe is itself dependent on the actions taken.

Following Ryall (2003), consider the pre-Columbian theory that the world was flat. Sailors thought that the world was flat and acted according to their available information by keeping their ships close to shore. However, by keeping close to shore, sailors never generated data that could refute their erroneous belief that they would sail off the face of the earth if they went too far from shore.

Self-confirming theories can be problematic in that the results generated are consistent with the results they predict. Moreover, predictions about counterfactual behaviors-what would have happened if the founders had taken some other course of action-are not observed (Ryall, 2003). Bayesian learning enhances subjective rationality in that it allows founders to update their initial beliefs based on the data that become available during the process. In the pre-Columbian example, sailors would update their beliefs about the earth based on Columbus's and Magellan's findings that the earth was round and not flat. The sailors would not continue to hold beliefs that were contrary to the incoming data.

The assumption of subjective rationality has been criticized by some experimental psychologists, who have shown that experimental subjects can behave contrary to Bayes' Rule (e.g., Kahneman & Tversky, 1973; Tversky & Kahneman, 1974). For example, it has been observed that individuals tend to give too much weight to observed "evidence" and, thus, too little weight to their prior beliefs (Grether, 1980: 554). Many behavioral decision researchers contend that people do not behave as Bayesians in practice, and they disagree with Hirshleifer and Riley's (1992) defense of subjective rationality on the grounds that individuals are likely to make more consistent and logical decisions when the stakes are higher.

Subjective rationality does not assume that individuals hold correct beliefs ex ante; indeed, it takes into account many biases that actors might have. Nor does subjective rationality assume anything about where people's priors come from. It recognizes that some people may hold beliefs that are very wide of the mark, such as our pre-Columbian sailors, and that these beliefs can take a long time to shift in accordance with the facts actors observe. Subjective rationality coupled with Bayesian learning helps capture the notion that individuals do not remain oblivious to factual information as data become available and known. Subjective rationality with Bayesian learning effectively strikes a middle course between behavioral decisionmaking theories and economic theories assuming full rationality.

To illustrate the use of Bayes' Rule in our model, consider, for the sake of expositional clarity, a tractable set of probability distributions to model both priors and data-those belonging to the normal family. Thus, suppose that the events x and y are drawn from the same normal distribution—x, y ~ $N(\mu, \phi)$, where μ denotes the central location of the normal distribution and ϕ denotes its variance. We assume that both founders are risk neutral and that any uncertainty centers on μ .⁸ The probability density function for some realization x_t depends on the unknown μ and is written as $p(\mathbf{x}_t|\mu) =$ $(2\pi\phi)^{-1/2} \exp\{-\frac{1}{2}(\mathbf{x}_t - \mu)^2/\phi\}$. Likewise, $p(\mathbf{y}_t|\mu) = (2\pi\phi)^{-1/2} \exp\{-\frac{1}{2}(\mathbf{y}_t - \mu)^2/\phi\}$.

Further, assume that the founders' priors are characterized by the distributions μ_A \sim N(μ_{A0} , ϕ_{A0}) and $\mu_B \sim \mathit{N}(\mu_{B0}, \ \phi_{B0})$. The corresponding prior probability density functions are $p_i(\mu) =$ $(2\pi\phi_{j0})^{-1/2}\exp\{-\frac{1}{2}(\mu-\mu_{j0})^2/\phi_{j0}\}, \text{ for } j=\{A, B\}.$ One can also precisely define the terms overoptimism and overconfidence by comparing these subjective priors with objective reality. A founder *j* is overoptimistic (or pessimistic) when μ_{i0} > (or <) μ . And a founder is more confident in his or her beliefs (i.e., his or her beliefs are more precise) the smaller the prior variance ϕ_{i0} . Overconfidence implies $\phi_{j0} < \phi$.⁹ For a sequence of data $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_t)$, (3)

becomes

$$p_{At}(\boldsymbol{\mu}|\mathbf{x}) \propto p_{A}(\boldsymbol{\mu}) \prod_{\tau=1}^{t} p(\mathbf{x}_{\tau}|\boldsymbol{\mu}), \qquad (4)$$

with an analogous expression for founder B replacing A with B and x with y. Substituting the various expressions for the $p(\cdot)$ terms given above into (4), Lee (1995: Chapter 2) shows that the posterior distributions of founders j also follow normal distributions, with means μ_{it} and variances ϕ_{it} , whose formulas are given in the first part of the Appendix.¹⁰ The key parameters of these distributions that bear on founders' decision making are μ_{At} and μ_{Bt} , which are A's and B's expected values of the true mean μ . The time subscript on these parameters shows that these values change over time as the founders change their beliefs using Bayesian updating.

An interesting by-product of Equation 4 is that it shows how initial beliefs, $p_i(\mu)$, become modified as more and more data arrive. Yesterday's posterior belief becomes today's prior belief, which is then modified by today's information to become today's posterior belief, and so the process goes on. Thus, priors are continually updated. In terms of the initial prior, for large enough t, the product term in Equation 4 (known as the complete likelihood) usually becomes the chief influence on posterior beliefs. We then say that the likelihood dominates the prior: even strongly held initial beliefs give way in the face of overwhelming objective evidence (Bereby-Meyer & Roth, 2006).

In general, when founders come together, they may have different prior beliefs, perhaps reflecting differences in their past experiences and interpretations of the world (Shafer, Nembhard, & Uzumeri, 2001). In conditions where there are

⁸ Allowing for uncertainty about ϕ as well would complicate the analysis considerably without changing the key results (e.g., see Campbell & Viceira, 2002, and Parker, 2007). The use of the normal distribution simplifies the exposition that follows; nothing essential depends on it. In particular, the unbounded negative outcomes it can generate pose no logical problem for entrepreneurial decision making.

⁹ It is notable that much of the entrepreneurship literature conflates the distinct concepts of overconfidence and overoptimism. This practice can lead to confusion and should be eschewed.

¹⁰ Note that the model does not require any assumption that the (x_t, y_t) draws be independent of each other. However, the derivation does assume that (x_t, y_t) are temporally independent. Allowing for autocorrelation would introduce extra nuisance parameters but would otherwise leave the analysis unchanged.

small amounts of information—a usual condition in which new firms are formed—founders' strong priors may continue to be the dominant influence on posterior beliefs. These differences in priors may lead to disagreements between the founders. Using the above arguments and Equation 4, this is most likely to occur when t is small—that is, when the prior dominates the likelihood. This, in turn, will have important implications for the allocation of decision-making rights in emerging firms.

The fact that priors can be different captures the notion that individuals can disagree. Precisely how these beliefs evolve depends on whether the founders operate in an environment characterized by *risk* or *uncertainty*, concepts we define in the next section.

For our purposes in this paper, the Bayesian approach is helpful in distinguishing between the concepts of risk and uncertainty (Cyert & DeGroot, 1987). Historically, the term *uncertainty* has been defined in many different ways, and it is beyond the scope of this paper to resolve these definitional issues. Here we define the terms *risk* and *uncertainty* in a way that is consistent with Knight (1921) and Foss and Klein (2005).

Founders seeking to organize firms face the same question examined by ICT: Who in the firm should own the residual rights of control? During the formation period, some founders have sufficient ex ante verifiable information about the potential set of outcomes of exploiting a market opportunity to reliably assign probability distributions to these outcomes, a setting described in this paper as one of risk (Demsetz, 1988).¹¹ Under conditions of risk, A and B have identical prior and posterior beliefs, and these prior and posterior beliefs are the same. That is, A and B both know the "true" objective distribution $N(\mu, \phi)$, and, knowing this, their beliefs do not change over time. This does not presume that A and B know which particular x and y values will arise; rather, they merely know the distribution from which the values are drawn.

Founders in risky conditions possess a wide range of decision-making tools to assign control rights (Milgrom & Roberts, 1992). Most economic and finance models of business decision making are applicable to risky decisions (Brealey & Myers, 1988; Cyert & DeGroot, 1987). For example, to calculate the present value of a new investment, both the possible outcomes associated with the investment and the probability of these outcomes must be known. These concepts find their analogues in the net cash flow an investment is expected to generate (i.e., possible investment outcomes) and the discount rate applied to that projected net cash flow (i.e., the likelihood that particular outcomes will occur).

In contrast, the decision to invest in a market opportunity is characterized as uncertain when the probabilities of outcomes are not known reliably ex ante at the time the decision is made (Foss et al., 2007; Foss & Klein, 2005; Knight, 1921; LeRoy & Singell, 1987).¹² That is, under uncertainty, A and B do not know the true distribution $N(\mu, \phi)$. Instead, they hold prior beliefs about it that generally differ from $N(\mu, \phi)$.¹³ In this case both founders are not only ignorant about which particular x and y values will arise at any time, but they are also ignorant about the distribution the values will be drawn from, having only a "best guess" represented by their prior beliefs. As noted above, A and B can hold completely different beliefs such that $(\mu_{A0}, \phi_{A0}) \neq (\mu_{B0}, \phi_{B0})$. Each founder knows the other's beliefs, but we assume this does not affect his or her own beliefs (i.e., priors are independent of each other). We also assume that no founder resorts to strategic reporting of his or her beliefs to the other founder. 14 To summarize so far, μ_{A0} and μ_{B0} (and ϕ_{A0} and ϕ_{B0}) can differ from each other and from

¹¹ Alternatively, as an anonymous reviewer pointed out, one can call this a situation of "known risk," whereas uncertainty as defined later is one of "unknown risk."

¹² An alternative possibility is that decision makers might not even know the set of outcomes, implying that the probabilities are unknown as well (Shackle, 1979) Arguably, however, this case resembles ambiguity (Einhorn & Hogarth, 1986) or ignorance, rather than uncertainty. We are grateful to an anonymous reviewer for pointing out this distinction.

¹³ Thus, we follow LeRoy and Singell's (1987) reading of Knight (1921), in which agents are always supposed to form subjective probability judgments and the risk-uncertainty distinction is identified with the distinction between objective and subjective probabilities.

¹⁴ Nonindependent beliefs and strategic reporting of beliefs would incorporate considerable extra complexity into the analysis. Regarding independence of beliefs, think of a theist and an atheist holding opposite prior beliefs about the existence of God. Being aware of the other's belief does not affect either's prior. Regarding strategic reporting, see the concluding discussion about a possible extension of the model that incorporates this feature.

the true μ (and ϕ) under uncertainty, but not under risk, where they all coincide.

It is helpful in what follows to distinguish between two cases of uncertainty. When founders start in conditions of uncertainty where information is plentiful, they have "strong" priors, defined as the case where the ϕ_{j0} are numerically small. As noted above, since the ϕ_{j0} measure the imprecision (or "spread") of beliefs, we can say that the founders are relatively confident about their beliefs. In this case the priors dominate the likelihood and confident founders tend to adjust their strong beliefs relatively slowly. In the limit, where $\phi_{j0} = 0$, individuals are said to possess "dogmatic" priors and never adjust their beliefs, irrespective of the available information.

In the second case founders start in conditions of uncertainty where information is scarce. They have "weak" priors, corresponding to high values of ϕ_{j0} , reflecting a lack of confidence in their beliefs. In this case the likelihood dominates the priors and founders adjust their beliefs very rapidly, in line with new information. Founders can still make investments, but an additional benefit of these investments is that they facilitate rapid acquisition of information, knowledge, and learning (Alvarez & Barney, 2007; Foss, 1996). Of course, regardless of which case applies in any context, founders still need decision-making tools to make well-founded decisions.

It is important to note at this juncture that whether or not a decision to invest in a market opportunity is risky or uncertain depends on the objective properties of that setting, not on the beliefs of decision makers (LeRoy & Singell, 1987: 398). No matter how a decision maker "feels" or what a decision maker "believes" about the outcomes of a decision, if the outcomes of a decision are not certain, then they are either risky or uncertain depending on the objective environment (March, 1991).

Figure 1 illustrates the different conditions of risk, uncertainty where founders have strong prior beliefs, and uncertainty where founders have weak prior beliefs. The distribution in the center of Figure 1 depicts the case of risk, where both A and B agree on the true distribution of outcomes. The distributions on the left and right illustrate the case of uncertainty where founders have strong priors. For example, if higher values



FIGURE 1 Risk, Uncertainty, and Prior Beliefs

along the horizontal axis represent objectively more favorable outcomes, then Figure 1 portrays B (whose predictive distribution about x outcomes, $p_{B}(x)$, lies to the right of the central curve) as being more optimistic than A,15 whereas A (whose predictive distribution, $p_A(x)$, lies to the left of the central curve) is more pessimistic than B. Bayesian updating predicts that as founders A and B learn from the data and update their beliefs, their predictive distributions shift from those illustrated in Figure 1 and converge on the true distribution $p(\mathbf{x}|\mu)$ shown in the center. Uncertainty where founders have weak prior beliefs is depicted by the relatively "flat" predictive distribution $p_{\rm C}(x)$, with a much higher variance $(\phi + \phi_{Ct})$ than any of those in Figure 1. As noted earlier, this case is associated with very limited confidence by a founder in his or her beliefs.

RESULTS

We now explore the implications for the organization of emerging firms. Firm formation under conditions of risk and uncertainty is each considered in turn. We then illustrate the model with a simple numerical example.

The second part of the Appendix shows that, under Bayesian learning, the condition (Equation 2) for replacing shared control of the firm with control by a single founder j can be written as

$$\theta_j(t) - \bar{\theta}(t) > \chi + \frac{\varphi_{t+1}}{2} \left(\mu_{j't} - \mu_{jt} \right), \tag{5}$$

where the $\theta_j(t)$ are given by Equation 1, where $\overline{\theta}(t) = \frac{1}{2}(\theta_A(t) + \theta_B(t))$ as noted above, and where μ_{jt} are given by Equation 7 in the Appendix. Equation 5 determines the allocation of control rights between the founders; it will turn out to be crucial for illustrating how risk differs from uncertainty.

The Case of Risk

The material below first restates a classic result from ICT before discussing some subtleties of the result in the context of new firm organization.

The first result follows by observing that, in the case of risk, both founders know and agree on the true distribution $N(\mu, \phi)$. So $\mu_{At} = \mu_{Bt} = \mu$ for all t. Hence, (5) simplifies to

$$\theta_i(t) - \bar{\theta}(t) > \chi.$$
 (6)

It follows directly that if (6) holds for any founder, it only holds for the most able founder who has the most to gain from being the residual claimant. We therefore have the first proposition.

> Proposition 1: Under conditions of risk, a founder who takes sole control of an emerging firm is the one with the most to benefit from the firm's payoffs.

While ICT has already established that the founder who benefits the most from residual rights of control will become the residual claimant (e.g., Grossman & Hart, 1987), there are two additional subtleties that arise in the specific context of new firm emergence. First, sole control by one founder is not inevitable. According to Equation 6, there will only be a single controller if the most productive founder at t has a productivity $\theta_i(t)$ that is sufficiently larger than the average $\theta(t)$ to compensate the other founder for the ensuing loss of control (the vanity bonus, χ). However, if the vanity bonus is too high, then Equation 6 does not hold for any founder and shared control ensues. Hence, sole control of new ventures will be more likely the smaller the vanity bonus.

Second, Equation 6 implies that emerging firms can exhibit a variety of control structures, which can change over time. If the equation holds for one founder, then the newly formed firm will be solely controlled by that founder. If it does not hold for either founder, then the newly formed firm will be organized under shared control. Since the founders' abilities change over time by Equation 1, the identity of the sole controller can therefore change as well.

The Case of Uncertainty

Under conditions of uncertainty, less reliable and verifiable information about the underlying distribution of outcomes is available than under conditions of risk (Knight, 1921; Simon, 1973). The

¹⁵ The predictive distribution $p_j(\mathbf{x}) = \int p_j(\mathbf{x}|\boldsymbol{\mu})p_j(\boldsymbol{\mu})d\boldsymbol{\mu}$ is j's current subjective prediction of the value of x, taking into account both the uncertainty about $\boldsymbol{\mu}$ and the residual uncertainty about x when $\boldsymbol{\mu}$ is known. The predictive distributions illustrated in Figure 1 are $N(\mu_{it}, \boldsymbol{\phi} + \phi_{it})$.

information that is available may be processed and interpreted in different ways by founders of early-stage firms. Recall that the difference between risk and uncertainty is that, under uncertainty, each founder has his or her own prior probability distribution, which generally differs from the prior distribution of others.

Under conditions of risk, the model is consistent with ICT's predictions about ownership structure. That is, the party to an exchange who has the most to gain should obtain residual rights of control to that exchange. However, under uncertainty, an additional layer of complexity is added, making it more difficult to reliably know, ex ante, the future value of specific investments in an exchange. Therefore, it becomes increasingly difficult to know who has the most to gain from these specific investments. When individuals are uncertain about the value of their specific investments and, thus, the value of residual rights of control, decisions about who should have these rights in an exchange may depend on the relative overoptimism of parties to that exchange (Sorenson & Waguespack, 2005).

Evidence suggests that many decision makers are systematically overoptimistic about their future prospects and that founders are especially prone to overoptimism (Camerer & Lovallo, 1999; Cooper, Woo, & Dunkelberg, 1988). For example, 68 percent of respondents to Cooper et al.'s (1988) survey of American entrepreneurs declared that the odds of their business succeeding were better than for others in the same sector, while only 5 percent thought that they were worse. Overoptimism entails inflated beliefs about success outcomes. From the earlier discussion, a founder *j* is more optimistic than *j*' if and only if $\mu_{jt} > \mu_{j't}$, overoptimism being defined as $\mu_{it} > \mu$. Nevertheless, overoptimism might be one of the reasons that entrepreneurs are willing to invest in uncertain settings in the first place (Busenitz & Barney, 1997).

If, over time, Bayesian learning enables founders to estimate the actual probability distribution of outcomes associated with an exchange, several outcomes are possible. One potential outcome is that the person who is allocated decisionmaking residual control rights actually is the ablest person to lead the firm. A firm organized on this basis may continue for some time. If, however, it turns out that this leader is not the ablest (but merely the most optimistic)—and, thus, from an objective standpoint of value maximization should no longer have residual control rightsseveral other outcomes may occur. It is possible that the overoptimistic leader may have exercised significant influence over others in the firm and will continue to lead the firm. This influence can ultimately reduce the level of the firm's performance and even jeopardize its survival. Alternatively, the overoptimistic leader may ultimately come to learn that he or she is no longer the best person to lead the firm and may agree to cede control to the more competent cofounder. The firm then can continue with the potential for higher productivity. Or there may be such disagreement that it cannot be resolved and the firm runs a greater risk of dissolution.

To explore these issues further, consider again Equation 5. Unlike the case of risk, founders operating under conditions of uncertainty generally have different prior beliefs, so $\mu_A \neq \mu_B$, and the term $\varphi_{t+1}/2(\mu_{i't} - \mu_{it})$ now influences the decision to allocate control rights. As in the case of risk, the allocation of control rights can change over time as information arrives, founders learn, and their productivities change. Hence, under uncertainty, control rights can switch between founders and between sole and shared control at different times, just as in the case of risk. But the key point is that, under uncertainty, true ability is no longer necessary and sufficient to determine who is allocated control rights, as was the case under risk with Equation 6. Instead, having a strong enough belief in one's own ability so that μ_{jt} exceeds $\mu_{j't}$ can be sufficient for j to secure j''s agreement to become the sole controller.

So, for example, if A is more optimistic about his or her own abilities than B, then that alone can be sufficient for A to seize control of the firm. This seems perhaps somewhat reminiscent of "charisma" theories of firm leadership, in which a charismatic leader has a stronger sense of selfbelief than others do of him/her and can convince the others of his or her preeminence, so becoming the leader by mutual consent. Charismatic leaders in complex organizations typically have a unique ability to transcend the bounds of everyday routines and are able under conditions of uncertainty to influence individual followers of the correctness of their vision. Charismatic leadership has been found to be more prevalent in conditions where the workforce or the market is rapidly changing or during changes in technology when bureaucratic or traditional rigidities are replaced

by teamwork (Quinn & Cameron, 1983; Robbins, 1983; Weber, 1903).

Unfortunately, sole control by the most overoptimistic founder might not be a good objective basis on which to organize a new firm. If B is actually less able than A, while being more optimistic about his or her ability than A is about his or her own ability (so $\mu_{Bt} > \mu_{At}$), then the firm would be more productive if A rather than B were in control. The trouble is that neither founder believes this change would be mutually beneficial. Hence, divergent priors under conditions of uncertainty can damage the interests of both firm founders.

To clarify, consider the special case where priors are so strong that they are dogmatic. They are associated with absolute and total confidence in one's own beliefs. In this case $\phi_{j0}=\phi_{j'0}=$ 0, and from the Bayesian updating formulas (7 and 8) given in the Appendix, it follows that $\mu_{At} = \mu_{A0}$ and $\mu_{Bt} = \mu_{B0}$ for all t. Thus, the founders' beliefs never change. So new firms that are organized on the basis of overoptimism rather than ability, where the less able but more optimistic founder takes sole control of the firm, can persist with this ownership structure for as long as the firm survives. Effectively, unshakable optimism is resistant to even mounting evidence of inferior ability. For the same reason, a sharing structure can persist for as long as the firm survives. As noted earlier, however, shared control is never optimal when founders possess different levels of ability, and this can put the firm at a competitive disadvantage, potentially jeopardizing its survival.¹⁶

These arguments are summarized in the following two propositions.

> Proposition 2: Under conditions of uncertainty, emerging firms can be organized with either sole or shared control, and control structures can change

over time. Inconsistent with ICT, the sole controller might no longer be the founder who objectively has the most to benefit from the exchange but, rather, the founder with the greatest optimism about his or her own potential benefits, who expects to benefit the most from running the firm. If beliefs are dogmatic, the identity of a controlling founder or a shared control structure never changes, for as long as the firm survives.

Proposition 3: When founders have dogmatic and different priors, initial conditions of uncertainty will lead to lower long-term firm output (and, hence, survival prospects) than will conditions of risk, except in the case when the abler founder possesses sufficient optimism to become the sole controller immediately.

Proposition 2 implies that the predictions of ICT may need to be extended for emerging firms under conditions of uncertainty, unlike the case of risk. And Proposition 3 implies that emerging firms under conditions of uncertainty may underperform emerging firms under conditions of risk.

When priors are not dogmatic (i.e., $0 < \phi_{A0}$ and $0 < \phi_{B0}$), the Bayesian updating formulas (7 and 8) given in the Appendix can be used to derive the result that $\lim_{t\to\infty} \mu_{At} = \lim_{t\to\infty} \mu_{Bt} = \mu$.¹⁷ That is, under uncertainty with nondogmatic priors, founders' beliefs change, and their expectations converge to the true mean of the data generation process.¹⁸ Expressed graphically, the two subjective probability distributions illustrated on the left and right sides of Figure 1 eventually move inward and converge onto the true probability distribution in the center. This limiting case is, of course, the one of risk assumed in traditional ICT. Thus, our theory of emerging firms dovetails into the traditional theory of the firm as those firms mature.

Although beliefs under uncertainty with nondogmatic priors converge with those under risk,

¹⁶ It might be thought that the vanity bonus χ serves a useful purpose in biasing firms' control structures toward efficient organizational forms. According to this argument, the larger the vanity bonus, the more convinced a founder has to be of his or her greater value to the firm before being willing to buy out the cofounder. Hence, the vanity bonus would seem to help prevent inefficient early buyouts by less able founders. But, in fact, the efficiency properties of the vanity bonus are not clear-cut. The vanity bonus promotes sharing outcomes, for any given abilities and beliefs. While this can prevent inefficient buyouts, it can clearly stymie efficient buyouts, too.

¹⁷ Note that this result does not require the use of a central limit theorem but follows directly from the structure of the posterior beliefs derived in the model.

 $^{^{18}}$ For any given sequence $\{\varphi_{\rm t}\},$ the larger ϕ_{A0} and $\phi_{B0},$ the faster this convergence is.

it does not follow automatically that actual control structures under uncertainty eventually converge with those applying under risk. If the weights $\{\varphi_t\}$ decline rapidly, early productivity draws quickly shape founders' long-run productivity. These can serve to lock early (and possibly inefficient) control structures into place for as long as the firm survives. Indeed, if inefficient control structures are locked into place for long enough, the emerging firm may not survive in any case. Hence, even nondogmatic priors can make uncertainty a debilitating environment for new entrepreneurial ventures.

There is one exception to this rule, which arises when uncertainty is so pronounced that founders have very weak prior beliefs. One might think that this must inevitably be an unfavorable environment for new entrepreneurial organizations to emerge in. In fact, an important counterintuitive insight yielded by the Bayesian modeling approach is that precisely the opposite turns out to be the case.

To understand why, consider a situation where founders' initial priors are different ($\mu_{A0} \neq \mu_{B0}$) and where individuals have very weak prior beliefs $(\phi_{A0} \text{ and } \phi_{B0} \text{ are both very large})$. In this situation both founders realize the uncertainty of their environment and, thus, lack confidence in their beliefs. New information then becomes the dominant influence on posterior beliefs such that the likelihood dominates the prior very rapidly; weak beliefs soon give way to reality in the face of the new information. This enables both founders to rapidly arrive at the same posterior distribution, despite having possibly very different initial priors.¹⁹ With the same priors, μ_{At} = μ_{Bt} , Equation 5 once again collapses into Equation 6. Founders' beliefs are the same, so they can reach the more efficient allocation of control rights, as in Proposition 1 for the case of risk.

Thus, we obtain the counterintuitive result that, from an objective standpoint, uncertainty where founders have weak prior beliefs can be a relatively benign context in which to operate firms, because founders more quickly come to agree on how to allocate control rights and avoid getting locked into inefficient control structures caused by overoptimism.

Proposition 4: Situations of uncertainty where founders commence with very imprecise incorrect prior beliefs are generally less damaging for founders than situations where incorrect priors are stronger. When founders are aware that their priors are imprecise and that they are indeed in conditions of uncertainty, these founders may be more open to learning and to adjusting their expectations based on this learning. Having founders who are more open to learning permits more efficient ownership structures to emerge and enhances firm survival.

Proposition 4 states that weak incorrect beliefs are superior, from an objective standpoint, to strong incorrect beliefs. Given the pervasive nature of overoptimism among entrepreneurs (Cooper et al., 1988), this seems to be a relevant point. Of course, it would always be objectively better still if both founders held strong correct beliefs.

Illustrative Example

Panel A of Table 1 describes a parameterization of the model that assigns specific numerical values for four key ingredients of the model: (1) the parameters of the normal distribution generating random events, (μ, ϕ) ; (2) the weighting scheme regulating how rapidly the impact of later events on founders' abilities decays, φ_t ; (3) the compensation both founders would need to cede control, χ ; and (4) four examples of initial priors of A and B, (μ_{A0}, ϕ_{A0}) and (μ_{B0}, ϕ_{B0}) .

Regarding (1), the true distribution generating x_t and y_t is taken to be normal, with mean $\mu = 2$ and variance $\phi = 1$. For (2), we use the simple form $\varphi_t = (1 + \rho)^{1-t}$, where higher values of ρ imply faster decay and so greater impacts on abilities from early events. Two different values of ρ are used. The first, $\rho = 0.1$, implies that early events mainly shape founders' abilities, while the second, $\rho = 0.5$, allows a greater role for later events to impact abilities as well. For (3), the cost of compensating a founder for losing control rights is $\chi = 0.5$. For (4), we consider four initial priors. Priors 1 and 2 are dogmatic for both founders ($\phi_{A0} = \phi_{B0} = 0$), and B is overop-

¹⁹ This can be seen formally by noting from the Bayesian updating formulas (7 and 8) given in the first part of the Appendix, that as ϕ_{A0} and ϕ_{B0} become larger, posterior expectations tend to $\mu_{A1} = \mu_{B1} = (x_1 + y_1)/2$, and $\mu_{At} = \mu_{Bt}$ for all t thereafter.

A. Parameters				
	(μ, ϕ) (2, 1)	$ec{oldsymbol{arphi}}_t \ (1 + ho)^{1-t}$	ρ (0.1, 0.5)	χ 0.5
	Priors 1	Priors 2	Priors 3	Priors 4
Π	(μ_{j0}, ϕ_{j0})	(μ_{j0}, ϕ_{j0})	(μ_{j0}, ϕ_{j0})	(μ_{j0}, ϕ_{j0})
$\begin{array}{l} \mathbf{A} \cdot \mathbf{x}_t \\ \mathbf{B} \cdot \mathbf{y}_t \end{array}$	(4.5, 0)	(1.23, 0)	(0.2, 2)	(20, 10)
		B. Output Realiza	lions	
			$\sum_{t}^{t} \mathbf{x}_{\tau} - \mathbf{y}_{\tau}$	$\sum_{\tau}^{t} \mathbf{x}_{\tau} - \mathbf{y}_{\tau}$
t	A: x,	B: <i>y</i> _t	$\sum_{r=1}^{2} \frac{1}{(1+0.1)^{1-\tau}}$	$\sum_{r=1}^{2} \frac{1}{(1+0.5)^{1-\tau}}$
1	2.49	0.07	2.42	2.42
2	1.60	2.53	1.57	1.80
3	-1.97	1.80	-1.54	0.12
4	0.17	2.28	-3.13	-0.50
5	1.80	2.75	-3.77	-0.69
6	2.29	1.79	-3.46	-0.62
		C. Control Struct	ure	
t, T	Priors 1, $\rho = 0.5$	Priors 2, $\rho = 0.5$	Priors 3, $\rho = 0.1$	Priors 4, $\rho = 0.1$
1	Share	A	A	Share
2	Share	A	A	Share
3	В	Share	A	В
4	В	Share	Share	В
5	В	Share	В	В
6	В	Share	В	В

TABLE 1 Numerical Example of Evolving Firm Structure

Note: In Panel B the sums correspond to twice the value of $\theta_A(t) - \overline{\theta}(t)$, which appears in equation reference {rc1}. In Panel C the column entries give the identity of the owner of control rights (A or B) if sole control rights are agreed upon, or indicate shared control rights (Share).

timistic while A is pessimistic. Priors 3 and 4 represent nondogmatic priors. With Priors 3, A is more overoptimistic than B, while Priors 4 illustrate the case where B is very overoptimistic ($\mu_{B0} = 20 > \mu = 2$) but lacks confidence ($\phi_{B0} = 10$).

Panel B lists six randomly generated realizations of x_t , y_t , and the relative individual productivity advantage of the founders, $\sum_{\tau=1}^t \varphi_{\tau}(x_{\tau} - y_{\tau}) = 2(\theta_A(t) - \overline{\theta}(t))$, for both values of ρ . A receives a favorable draw at t = 1 of $x_1 = 2.49$, but in subsequent periods B tends to receive better draws than A, obtaining, for example, $y_2 = 2.53$ compared with $x_2 = 1.60$ in period 2. Depending on the value of ρ , B's productivity overtakes A's by the third or fourth period. Panel C summarizes the control structure of the new firm under different values of ρ . These outcomes are based on the use of Equation 5.

This example illustrates the following points. Priors I represent a case where both A and B hold dogmatic beliefs. A is more productive than B in the first two periods, but B is more optimistic about his or her own ability. These two effects cancel out, and shared ownership results. But by period 3 A's productivity advantage has eroded, which, together with B's greater optimism, leads B to become the sole controller. In subsequent periods B becomes more productive than A, and this control structure becomes entrenched. A similar pattern is observed in the final column of Panel C, where beliefs are not dogmatic but B is highly overoptimistic while A is pessimistic.

Like Priors 1, Priors 2 are dogmatic, but now the degree of B's overoptimism is less pronounced than with Priors 1. As before, A is more productive than B in the first two periods, but this is now sufficient to give A sole control (see column 2 of Panel C). In subsequent periods B becomes more productive, but neither this advantage nor B's optimism is sufficient to wrest sole control from A. The outcome from period 3 is shared control. This case illustrates a permanent inefficient sharing structure, which could put the new firm at a competitive disadvantage.

Priors 3 show what happens when priors are strong but not dogmatic. A is more optimistic than B, takes over the firm immediately, and continues in sole control until period 4, when B's greater productivity offsets this and leads to shared control, followed by sole control by B from period 5 onward. This illustrates the interesting case where several different control structures are observed early in the life of the new venture.²⁰ Effectively, as A's priors become dominated by the data, A eventually gives way, and the more efficient outcome of sole control by B occurs. This illustrates the value of nondogmatic priors under conditions of uncertainty.

DISCUSSION

Using Bayesian modeling to analyze the allocation of residual rights of control under conditions of risk, uncertainty with weak prior beliefs, and uncertainty with strong prior beliefs generates new theoretical insights. We discuss some of these below, together with some suggestions for future research.

Emergence, Risk, and Uncertainty

The arguments developed in this paper depend on the distinction between risk and uncertainty. Current ICT applies quite well under conditions of risk, but our results suggest that these theories need to be extended and modified under conditions of uncertainty. For example, when founders have weak prior beliefs under uncertainty, the lack of information in this environment may actually make learning more feasible and may result in a faster allocation of residual rights of ownership. However, when founders have strong prior beliefs under uncertainty, there may be just enough information that the founders believe they can assign probabilities with reliability and have firm but different perceptions of what the potential outcomes might be. Couple radically different perceptions with overoptimism and it becomes clear why some firms end prematurely because of disagreements.

Bayesian learning models are particularly fruitful for management decision making in these different contexts and may be applied beyond the case of emerging firms. The analysis here has been for the particular case of emerging firms, in which uncertainty is especially prevalent, but it seems likely that conditions of uncertainty may apply at certain times in established organizations too. Managers in these firms may face uncertain conditions in which they are unable to use standard tools that are applicable to conditions of risk. Hence, the insights generated by a Bayesian learning approach might be useful for organizational scholars studying organizational decision making in more established firms as well.

For example, managers of incumbent firms may be interested in joining strategic alliances (Kogut, 1991) or initiating new corporate spinoffs (Lockett, Siegel, Wright, & Ensley, 2005), which entail entering new markets or innovating new products. The returns from and costs of these activities are likely to be uncertain rather than risky, since reliable information about new markets and products is likely to be very limited. Even existing markets might change from being risky to being uncertain-for example, in the case of disruptive technologies (Christensen, Anthony, & Roth, 2004), such as the telephone, minimills, and so forth. There is also the case of high-velocity industries (Eisenhardt, 1989), where current and historical information may not be useful owing to the rapidly changing environment. Whether the firm is emerging or an incumbent in an industry undergoing change, these industry changes may require different ways of evaluating new and changing information. Using conventional perspectives for decision making, such as net present value analysis, may well be efficient in established

²⁰ In fact, frequent switches of control might entail organizational problems of their own. We do not explore this issue further here.

industries. However, in industries that are emerging or undergoing a transition, Bayesian learning models may give managers a more complete understanding of their own biases and processes in navigating their environments.

Entrepreneurial Opportunity Formation and Exploitation

Another contribution of this paper is to emphasize that not all emerging firms face the same conditions, and these differing conditions make it necessary for firms to approach business processes such as planning and decision making differently. It seems reasonably clear that a variety of business process techniques that assume the underlying distribution of outcomes is, in principle, known are appropriate under conditions of risk. They may apply less well under conditions of uncertainty, where Bayesian learning models are likely to be more fruitful.

This paper suggests a potential need for entrepreneurship scholars to develop a typology of emerging entrepreneurial firms and to further understand dimensions of these organizations. In the current analysis we try to understand emerging firms with regard to the actions of founders (Alvarez & Barney, 2007) and to show how these different actions affect the types of decision-making models that can be used (Baker & Nelson, 2005; Sarasvathy, 2001). That these differing actions may result in different types of emerging firms suggests that there are also other characteristics of these firms that vary. Understanding variations between such firms may be a fruitful and rich area of research.

Moreover, our findings are not necessarily consistent with Shane and Venkataraman's (2000). These authors assume that entrepreneurial opportunities are homogeneous in their epistemological origins. While Shane and Venkataraman acknowledge that opportunities can be exploited either through the market or through a firm, most opportunities are exploited through the use of a firm, yet the authors remain silent about the use of a firm to exploit opportunities. The kind of firm that emerges may depend on the epistemological origins and ontology of the opportunity. This silence leaves the reader to assume that the only distinction of opportunity exploitation is that of hierarchy or market, not the possibility that firms exploiting opportunities might vary. The firms in our paper are quite heterogeneous, suggesting that the heterogeneous origins of opportunities might result in pronounced variations in firm structures and outcomes. The different decision-making processes explored here might be an indication that sources and types of information and knowledge also vary among entrepreneurial opportunities and among firms exploiting these opportunities. This suggests a need for further entrepreneurship research linking the epistemological origins and ontology of opportunities and their formation with variations in firm structure and performance.

A third insight of this paper is that, during firm formation, organizational structures may be unstable, as the founders update their beliefs and develop their abilities. However, the likely instability and temporariness of these entrepreneurial forms of organization does not discount their importance. Indeed, without these often temporary emerging firms, it is unlikely that the resources required to exploit a market opportunity will be brought together and coordinated in the first place. In this sense, the act of organizing a firm under uncertainty may help create the opportunities that founders are then able to exploit.

Moreover, without these firms, uncertainty is unlikely to ever evolve into risk, since there will be no coordinated resources brought together to try to exploit market opportunities. Without this initial coordination of resources, information about the probability distribution of outcomes associated with an exchange may never become known. In this setting the potential value of exploiting an opportunity-to investors and to society more broadly-may not be realized. Thus, while emerging firms may be thought of as institutional "place holders" until the most efficient form of organization under risk can be learned, they are essential to the process of changing uncertainty into risk and, thus, essential to the process of learning the most efficient way to manage a particular set of economic exchanges.

This more nuanced approach to the potential failure and benefits of failure of these emerging firms provides a more realistic view of the process of firm formation. Indeed, one possible explanation for these emerging firms may be a real options approach (McGrath, 1999). However, instead of the traditional option value being realized by an individual firm, it may, in fact, be that the option value is realized by society in general as more is learned about the innovation and the underlying conditions that an emerging firm is operating in.

In conclusion, the foregoing discussion suggests that the model developed in this paper carries several implications for future research in organizational behavior and entrepreneurship. We contend that models of the type proposed here can fructify the field of entrepreneurship by providing a more systematic treatment of decision making under conditions resembling what firm founders typically encounter in practice. Nevertheless, we acknowledge that our model represents only a starting point in this respect. Several extensions hold the promise of generating further insights germane to entrepreneurial and managerial decision making and organizational structure.

For example, our model assumes that individual founder productivity evolves only stochastically, without purposive investment by founders to enhance their own performance and that of the firm. It would be desirable in future research to relax this assumption by introducing forwardlooking strategic investment considerations. A natural approach to take would be analysis of a dynamic optimization problem with endogenous investment behavior. Founders would optimize subject to their prior beliefs; their investments would be based on those beliefs and would lead to outcomes generating new data from which they would learn and make new investment decisions. Whether or not these outcomes would validate prior beliefs and lead to efficient or inefficient equilibria is unclear a priori (see Ryall, 2003, for a discussion in the context of oligopolistic competition between incumbents). It seems entirely possible that Bayesian founders could get stuck in self-confirming equilibria in an extended model of this sort. In that context it would be interesting to know whether uncertainty is a more or less favorable environment than risk, paralleling the questions explored in this paper.

A second extension might introduce strategic interactions between new venture founders. This paper has explored the implications of disagreements between founders caused by divergent prior beliefs. At least one of the new software ventures cited in the second footnote of this article can be thought of in these terms. However, there might be other possibilities too. For example, one can imagine that instead of revealing information about abilities and beliefs truthfully to cofounders, some "unscrupulous" founders might choose to conceal information and to engage in a strategic game with their partners for personal advantage. Thus, the model could be extended along the lines of a "cheap talk" game to allow for strategic reporting of abilities (Battaglini, 2002; Crawford & Sobel, 1982). It seems likely that this could provide an alternative reason why shared control often comes to be replaced by sole control in new ventures: clever and ambitious founders force out their cofounders by engaging in skillful but self-serving pursuit of strategic advantage. Other questions then immediately arise about the efficiency of these outcomes compared with situations without strategic elements, as well as the scope of clever contracting to restrict opportunism in games of this sort. These questions clearly take us far beyond the reaches of the present article and probably require separate papers of their own.

The applicability of Bayesian learning models to characterize decision making is probably far broader than our study has suggested. We hope that future researchers will start to apply Bayesian modeling to the decision-making processes of established firms as well, to better explain a variety of organizational outcomes. These might include not only ownership structure, explored here, but also decisions about a range of strategic variables, such as product development, alliances, and acquisitions.

APPENDIX

Bayesian Updating

By time t, each founder j's beliefs are represented by a normal posterior distribution with parameters

$$\mu_{jt} = \phi_{jt} \left[\frac{\mu_{j0}}{\phi_{j0}} + \frac{1}{\phi} \sum_{\tau=1}^{t} (\mathbf{x}_{\tau} + \mathbf{y}_{\tau}) \right] \quad \text{and} \qquad (7)$$

$$\phi_{jt} = \left(\frac{1}{\phi_{j0}} + \frac{2t}{\phi}\right)^{-1} \tag{8}$$

for each $j \in \{A, B\}$ (see Lee, 1995: Chapter 2, for the proof).

Derivation of the Control Allocation Rule (Equation 5)

The expectations in Equation 2 can be written in terms of the data by using Equation 1:

$$E_{jt}[\theta_{j}(t+1)] = E_{jt}\left[\sum_{\tau=1}^{t+1} \varphi_{\tau} \mathbf{x}_{\tau}\right] = E_{jt}\left[\sum_{\tau=1}^{t} \varphi_{\tau} \mathbf{x}_{\tau} + \varphi_{\tau+1} \mathbf{x}_{\tau+1}\right] = \theta_{j}(t) + \varphi_{\tau+1} \mu_{jt}$$

$$E_{j't}[\theta(t+1)] = \theta(t) + \varphi_{\tau+1}\mu_{j't}$$

 $E_{jt}[\bar{\theta}(t+1)] = E_{jt}\left[\frac{1}{2}\sum_{\tau=1}^{t+1}\varphi_{\tau}(\mathbf{x}_{\tau}+\mathbf{y}_{\tau})\right]$

 $=ar{ heta}(t)+arphi_{ au+1}\mu_{jt}$

Putting each of these three expressions into Equation 2 yields the condition

$$egin{aligned} & heta_j(t) > -arphi_{ au+1}\mu_{jt} + \chi + rac{1}{2}igl[ar{ heta}(t) + ar{ heta}(t) + arphi_{ au+1}(\mu_{jt} + \mu_{j't})igr] \ &= \chi + ar{ heta}(t) + rac{1}{2}arphi_{ au+1}(\mu_{jt} + \mu_{j't}) - arphi_{ au+1}\mu_{jt}, \end{aligned}$$

from which Equation 5 follows directly.

REFERENCES

- Alchian, A. A., & Demsetz, H. 1972. Production, information costs, and economic organization. *American Economic Review*, 62: 777–795.
- Alvarez, S. A., & Barney, J. 2005. How entrepreneurs organize firms under conditions of uncertainty. *Journal of Man*agement, 31: 776–793.
- Alvarez, S. A., & Barney, J. B. 2007. Discovery and creation: Alternative theories of entrepreneurial action. *Strategic Entrepreneurship Journal*, 1(1–2): 11–26.
- Argote, L. 1999. Organizational learning: Creating, retaining and transferring knowledge. Norwell, MA: Kluwer.
- Argote, L., Beckman, S. L., & Epple, D. 1990. The persistence and transfer of learning in industrial settings. *Management Science*, 36: 140–154.
- Baker, T., & Nelson, R. 2005. Creating something from nothing: Resource construction through entrepreneurial bricolage. Administrative Science Quarterly, 50: 329–366.
- Battaglini, M. 2002. Multiple referrals and multidimensional cheap talk. *Econometrica*, 70: 1379–1401.
- Bereby-Meyer, Y., & Roth, A. 2006. The speed of learning in noisy games: Partial reinforcement and the sustainability of cooperation. *American Economic Review*, 96: 1029– 1042.

- Brealey, R., & Myers, S. 1988. Principles of corporate finance. New York: McGraw-Hill.
- Bruton, G., Fried, R. D., & Hisrich, R. D. 1997. Venture capitalist and CEO dismissal. *Entrepreneurship: Theory & Practice*, 21(3): 41–54.
- Buckley, P., & Michie, J. 1996. Firms, organizations, and contracts. Oxford: Oxford University Press.
- Busenitz, L. W., & Barney, J. B. 1997. Differences between entrepreneurs and managers in large organizations: Biases and heuristics. *Journal of Business Venturing*, 12: 9–30.
- Camerer, C., & Lovallo, D. 1999. Overconfidence and excess entry: An experimental approach. *American Economic Review*, 89: 306–318.
- Campbell, J. Y., & Viceira, L. M. 2002. Strategic asset allocation: Portfolio choice for long-term investors. Oxford: Oxford University Press.
- Christensen, C. M., Anthony, S. D., & Roth, E. A. 2004. Seeing what's next. Boston: Harvard Business School Press.
- Cooper, A. C., Woo, C. Y., & Dunkelberg, W. C. 1988. Entrepreneurs' perceived chances for success. *Journal of Business Venturing*, 3(6): 97–108.
- Crawford, V., & Sobel, J. 1982. Strategic information transmission. *Econometrica*, 50: 1431–1451.
- Cyert, R. M., & DeGroot, M. H. 1987. Bayesian analysis and uncertainty in economic theory. Totowa, NJ: Rowman and Littlefield.
- Cyert, R. M., & March, J. G. 1992. A behavioral theory of the firm (2nd ed.). Cambridge: Blackwell.
- Demsetz, H. 1988. Profit as a functional return: Reconsidering Knight's views. Oxford & New York: Blackwell.
- Dosi, G., Nelson, R., & Winter, S. 2000. The nature and dynamics of organizational capabilities. Oxford: Oxford University Press.
- Einhorn, H. J., & Hogarth, R. M. 1986. Decision making under ambiguity. *Journal of Business*, 59(Supplement): 225–250.
- Eisenhardt, K. M. 1989. Making fast strategic decisions in high-velocity environments. Academy of Management Journal, 32: 543–576.
- Foss, K., Foss, N., & Klein, P. G. 2007. Original and derived judgment: An entrepreneurial theory of economic organization. Organization Studies, 28(12): 1–20.
- Foss, N., & Klein, P. G. 2005. Entrepreneurship and the theory of the firm: Any gains from trade? In S. A. Alvarez, R. Agarwal, & O. Sorenson (Eds.), *Handbook of entrepreneurship: Disciplinary perspectives*, vol. 2: 55–80. Berlin: Springer.
- Foss, N. J. 1996. Firms, incomplete contracts, and organizational learning. *Human Systems Management*, 15: 17–26.
- Foss, N. J. 2000. The theory of the firm: Critical perspectives on business and management. London: Routledge.
- Grether, D. M. 1980. Bayes Rule as a descriptive model: The representativeness heuristic. Quarterly Journal of Economics, 95: 537–557.
- Grossman, S., & Hart, O. 1986. The costs and benefits of

ownership: A theory of vertical integration. *Journal of Political Economy*, 94: 691–719.

- Grossman, S., & Hart, O. 1987. One share-one vote and the market for corporate control. *Journal of Financial Economics*, 20: 175–202.
- Haas, M. R. 2006. Knowledge gathering, team capabilities, and project performance in challenging work environments. *Management Science*, 52: 1170–1184.
- Hart, O., & Moore, J. 1988. Incomplete contracts and renegotiation. *Econometrica*, 56: 755–786.
- Hirshleifer, J., & Riley, J. G. 1992. The analytics of uncertainty and information. Cambridge & New York: Cambridge University Press.
- Huber, G. P. 1991. Organizational learning: The contributing processes and the literatures. **Organization Science**, 2: 88–115.
- Kahneman, D., & Tversky, A. 1973. On the psychology of prediction. *Psychological Review*, 80: 237–251.
- Kalnins, A., Swaminathan, A., & Mitchell, W. 2006. Turnover events, vicarious information, and the reduced likelihood of outlet-level exit among small multiunit organizations. Organization Science, 17: 118–131.
- Kauffman Center for Entrepreneurial Leadership. 1999. Research report. Kansas City, MO: Kauffman Center for Entrepreneurial Leadership.
- Knight, F. 1921. Risk, uncertainty and profit. New York: Houghton Mifflin.
- Kogut, B. 1991. Joint ventures and the option to expand and acquire. *Management Science*, 37: 19–33.
- Lee, P. M. 1995. Bayesian statistics: An introduction (2nd ed.). London: Arnold.
- Lerner, J. 1995. Venture capitalists and the oversight of private firms. *Journal of Finance*, 50: 301–318.
- LeRoy, S. F., & Singell, L. D. 1987. Knight on risk and uncertainty. Journal of Political Economy, 95: 394–406.
- Lockett, A., Siegel, D., Wright, M., & Ensley, M. D. 2005. The creation of spin-off firms at public research institutions: Managerial and policy implications. *Research Policy*, 34: 981–993.
- March, J. G. 1991. Exploration and exploitation in organizational learning. Organization Science, 2: 71–87.
- McGrath, R. G. 1997. A real options logic for initiating technology positioning investments. Academy of Management Review, 22: 974–996.
- McGrath, R. G. 1999. Falling forward: Real options reasoning and entrepreneurial failure. Academy of Management Review, 24: 13–30.
- Milgrom, P., & Roberts, J. 1992. *Economics, organization and management*. New York: Prentice-Hall.
- Mosakowski, E. 1997. Strategy making under causal ambiguity: Conceptual issues and empirical evidence. **Orga***nization Science*, 8: 414–442.

- Parker, S. C. 2007. Entrepreneurial learning and the existence of credit markets. *Journal of Economic Behavior &* Organization, 62: 37–46.
- Quinn, R. E., & Cameron, K. 1983. Organizational life cycles and shifting criteria of effectiveness. Some preliminary evidence. *Management Science*, 29: 33–52.
- Reagans, R., Argote, L., & Brooks, D. 2005. Individual experience and experience working together: Predicting learning rates from knowing who knows what and knowing how to work together. *Management Science*, 15: 869–882.
- Robbins, S. P. 1983. The theory Z organization from a powercontrol perspective. *California Management Review*, 25(2): 67–75.
- Ryall, M. D. 2003. Subjective rationality, self-confirming equilibrium, and corporate strategy. *Management Science*, 49: 936–949.
- Sarasvathy, S. D. 2001. Causation and effectuation: Toward a theoretical shift from economic inevitability to entrepreneurial contingency. Academy of Management Review, 26: 243–263.
- Shackle, G. L. S. 1979. *Imagination and the nature of choice*. Edinburgh: Edinburgh University Press.
- Shafer, S. M., Nembhard, D. A., & Uzumeri, M. V. 2001. The effects of worker learning, forgetting, heterogeneity on assembly line productivity. *Management Science*, 47: 1639–1653.
- Shane, S., & Venkataraman, S. 2000. The promise of entrepreneurship as a field of research. Academy of Management Review, 25: 217–226.
- Simon, H. A. 1973. The structure of ill structured problems. Artificial Intelligence, 4: 181–201.
- Sorenson, O., & Waguespack, D. 2005. Social networks and exchange: Self-confirming dynamics in Hollywood. Mimeograph, University of California at Los Angeles.
- Tversky, A., & Kahneman, D. 1974. Judgement under uncertainty: Heuristics and biases. Science, 185: 1124–1131.
- Ucbasaran, E., Lockett, A., Wright, M., & Westhead, P. 2003. Entrepreneurial founder teams: Factors associated with member entry and exit. *Entrepreneurship: Theory & Practice*, 28(2): 107–127.
- Wadeson, N. 2004. Multi-dimensional search: Choosing the right path. International Journal of the Economics of Business, 11: 287–301.
- Weber, M. 1903. *The methodology of the social sciences*. New York: Free Press.
- Williamson, O. E. 1975. Markets and hierarchies: Analysis and antitrust implications. New York: Free Press.
- Zahra, S. A., Ireland, R. D., & Hitt, M. A. 2000. International expansion by new venture firms: International diversity, mode of market entry, technological learning, and performance. Academy of Management Journal, 43: 925–950.

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